Wasserstoffatom

$$\left\{ \frac{d^2}{d r^2} + \frac{2}{\rho}\frac{d}{d\rho} - \frac{l(l+1)}{\rho^2} + \frac{N}{\rho} - \frac{1}{4} \right\} R(\rho) = \emptyset$$

$$R(\rho) = \sum_{k=0}^{\infty} c_k \, \rho^{k+\alpha}$$

$$\sum_{k=0}^{\infty} c_k \Big[(k+\alpha)(k+\alpha-1)\, \rho^{k+\alpha-2} + \frac{2}{\rho}(k+\alpha)\, \rho^{k+\alpha-1}$$

$$- \frac{l(l+1)}{\rho^2}\, \rho^{k+\alpha} + \frac{N}{\rho}\, \rho^{k+\alpha} - \frac{1}{4}\, \rho^{k+\alpha} \Big] = \emptyset$$

Terme von $k = \emptyset$:

$$c_0 \left[\alpha(\alpha-1)\, \rho^{\alpha-2} + 2\alpha\, \rho^{\alpha-2} - l(l+1)\, \rho^{\alpha-2} \right] = \emptyset$$

$\rho^{\alpha-1}$ & ρ^{α} Ordnungen werden weggelassen,

weil $\rho \to \emptyset$ angeschaut wird

$$\rightarrow \quad \alpha(\alpha-1) + 2\alpha - l(l+1) = \emptyset$$

$$\alpha^2 - \alpha + 2\alpha - \text{—"—} = \emptyset$$

$$\alpha^2 + \alpha - \text{—"—} = \emptyset$$

$$\rightarrow \quad \alpha = \begin{cases} l \\ -l-1 \end{cases} \qquad R = \rho^{\alpha}$$

$$= \rho^{l} \;\; || \;\; \rho^{-l-1}$$

bei $\rho \to 0$ wäre 2. Fall ∞ ?? $\rightarrow$ irregulär

$$R(\rho) = A\, \rho^{\ell}\, L(\rho)\, e^{-\frac{1}{2}\rho}$$

$$\left\{ \frac{d^2}{d\rho^2} + \frac{2}{\rho}\frac{d}{d\rho} - \frac{\ell(\ell+1)}{\rho^2} + \frac{N}{\rho} - \frac{1}{4} \right\} R(\rho) = \emptyset$$

Ansdrom Skriptum

$$\frac{1}{\hbar}\, \vec{\sigma}\vec{p}\, \psi_B = -\left(\varepsilon_-^{\,2} - \frac{Z\alpha\rho}{r} \right) \psi_A$$

$$\vec{\sigma}\vec{p} = \frac{1}{r^2}(\vec{\sigma}\vec{r})\left(-i\hbar r\partial_r + i\vec{\sigma}\vec{L} \right)$$

$$\frac{1}{\hbar}\,\frac{1}{r^2}(\vec{\sigma}\vec{r})\left(-i\hbar r\partial_r + i\vec{\sigma}\vec{L} \right)\psi_B = -\left(\text{— " —} \right)\psi_A$$

$$\psi_B = i\frac{g}{r}\, \exp(-\varepsilon_+\varepsilon_- r)\; \mathcal{Y}_{jm\ell_B}$$

$$\frac{1}{\hbar}\,\frac{1}{r^2}(\vec{\sigma}\vec{r})\left(-i\hbar r\partial_r + i\vec{\sigma}\vec{L} \right) \frac{ig}{r}\, e^{-\varepsilon_+\varepsilon_- r}\, \mathcal{Y}_{jm\ell_B} = \text{— " —}$$

$$-i\hbar r\partial_r\left(\frac{ig}{r}\, e^{-\varepsilon_+\varepsilon_- r} \right) = \hbar r\, e^{-\varepsilon_+\varepsilon_- r}\, \partial_r \frac{g}{r}$$

$$+ \hbar r \frac{g}{r}\, \partial_r\, e^{-\varepsilon_+\varepsilon_- r}$$

$$= \hbar r\, e^{-\varepsilon_+\varepsilon_- r}\left(\frac{1}{r}\partial_r g - \frac{g}{r^2} \right)$$

$$+ \hbar r \frac{g}{r}\left(-\varepsilon_+\varepsilon_- \right) e^{-\varepsilon_+\varepsilon_- r}$$

$$= \hbar \cancel{r}\, e^{-\varepsilon_+\varepsilon_- r}\left[\frac{1}{\cancel{r}}\partial_r g - \frac{g}{r^{\cancel{2}}} - \varepsilon_+\varepsilon_- \frac{g}{\cancel{r}} \right]$$

$$\frac{1}{\hbar}\,\frac{1}{r^2}(\vec{\sigma}\vec{r})\left[\hbar\, e^{-\varepsilon_+\varepsilon_- r}\, \mathcal{Y}_{jm\ell_B}\left(\partial_r g - \frac{g}{r} - \varepsilon_+\varepsilon_- g \right) \right.$$

$$+ i \vec{\sigma}\vec{L} \quad i \frac{f}{r} e^{-\varepsilon_+\varepsilon_- r} Y_{jml_B} \Big] = \dots$$

$$\| \; (\vec{\sigma}\vec{r})(\vec{\sigma}\vec{L}) = \vec{r}\vec{L} + i\,\vec{\sigma}(\vec{r}\times\vec{L})$$

$$\frac{1}{r}(\vec{\sigma}\vec{r}) \; Y_{jml_B} = - Y_{jml_B}$$

Sprawdzenie: [illegible]

$$\left(\frac{df}{dr} - \varepsilon_+\varepsilon_- f + \nu\frac{j+\frac{1}{2}}{r} f\right) = \left(\varepsilon_+^2 + \frac{Z\alpha_f}{r}\right) g$$

$$f(r) = \sum a_k r^{k+q} \qquad g(r) = \sum b_k r^{k+q}$$

$$\sum a_k\left((k+q)\, r^{k+q-1} - \varepsilon_+\varepsilon_- r^{k+q} + \nu\left(j+\tfrac{1}{2}\right) r^{k+q-1}\right)$$

$$= \sum b_k\left(\varepsilon_+^2 r^{k+q} + Z\alpha_f r^{k+q-1}\right)$$

f. $k = 0$:

1. $q + \nu\left(j + \frac{1}{2}\right) = Z\alpha_f$

2. $q - \nu\left(j + \frac{1}{2}\right) = -Z\alpha_f$

$$q = -\left(\nu\left(j+\tfrac{1}{2}\right) - Z\alpha_f\right)$$

$$q = \left(\nu\left(j+\tfrac{1}{2}\right) - Z\alpha_f\right)$$

$$q_1 = a - b \qquad (a+b)(a-b) = a^2 - b^2$$

$$q_2 = a + b \qquad q_1 q_2 = a^2 - b^2 \qquad ??$$

$(4.85) \cdot \varepsilon_-$

$$\left[(k+1+q) + \nu\left(j+\tfrac{1}{2}\right)\right] a_{k+1}\,\varepsilon_- - Z\alpha\rho\, b_{k+1}\,\varepsilon_- = \varepsilon_+ \varepsilon_-^2 a_k + \varepsilon_+^2 \varepsilon_- b_k$$

$(4.86)\ \varepsilon_+$

$$Z\alpha\rho\, a_{k+1}\varepsilon_+ + \left[(k+1+q) - \nu\left(j+\tfrac{1}{2}\right)\right] b_{k+1}\,\varepsilon_+ = \varepsilon_-^2 \varepsilon_+ a_k + \varepsilon_+^2 \varepsilon_- b_k$$

$$\overbrace{\left[(k+1+q) + \nu\left(j+\tfrac{1}{2}\right)\right]}^{c_{1+}} a_{k+1}\,\varepsilon_- - \overbrace{Z\alpha\rho}^{c_{2+}}\, b_{k+1}\,\varepsilon_-$$

$$= \overbrace{\left[(k+1+q) - \nu\left(j+\tfrac{1}{2}\right)\right]}^{c_{3+}} b_{k+1}\,\varepsilon_+ + \overbrace{Z\alpha\rho}^{c_{4+}}\, a_{k+1}\,\varepsilon_+$$

$$k+1 \to k$$

$$c_1 a_k - c_2 b_k = c_3 b_k + c_4 a_k$$

$$c_1 a_k - c_4 a_k = c_3 b_k + c_2 b_k$$

$$a_k (c_1 - c_4) = b_k (c_3 + c_2)$$

$$C_k = \frac{a_k}{c_3 + c_2} = \frac{b_k}{c_1 - c_4} \quad \checkmark$$

$$C_{k+1} = \frac{a_{k+1}}{c_{3+} + c_{2+}} = \frac{b_{k+1}}{c_{1+} - c_{4+}}$$

$$C_k = \frac{a_k}{\left[(k+q) - \nu\left(j+\tfrac{1}{2}\right)\right]\varepsilon_+ + Z\alpha\rho\,\varepsilon_-}$$

$$c_{n+1} = \frac{a_{n+1}}{\underbrace{\left[(k+\rho+1) - \nu\left(j+\tfrac{1}{2}\right)\right]\varepsilon_+ + Z\alpha_f\,\varepsilon_-}_{D}}$$

$$a_{n+1} \to b_{n+1}\ \&\ a_n\ \&\ b_n$$

$$c_1 a_{n+1} - c_2 b_{n+1} = c_3 a_n + c_4 b_n \quad | \cdot d_2$$

$$d_1 a_{n+1} + d_2 b_{n+1} = d_3 a_n + d_4 b_n \quad | \cdot c_2$$

$$\underbrace{c_1 d_2 a_{n+1} + d_1 c_2 a_{n+1}} = \underbrace{(c_3 d_2 + d_3 c_2)} a_n + (c_4 d_2 + d_4 c_2) b_n$$

$$\underbrace{(c_1 d_2 + d_1 c_2)}_{c_7} a_{n+1} = c_5 a_n + c_6 b_n$$

$$a_{n+1} = \frac{c_5 a_n + c_6 b_n}{c_7}$$

$$c_{n+1} = \frac{c_5 a_n + c_6 b_n}{c_7 D}$$

a_n & b_n durch c_n ersetzen!!

✓

$$2\varepsilon_+\varepsilon_-(n+q) - Z\alpha_f(\varepsilon_+^2 - \varepsilon_-^2) = 0$$

$$\varepsilon_\pm = \sqrt{\frac{m_0 c^2 \pm E}{\hbar c}}$$

$$2\sqrt{\frac{(m_0c^2+E)(m_0c^2-E)}{\hbar^2 c^2}}\,(n+q) - Z\alpha_f\left(\frac{m_0c^2+E}{\hbar c} - \left(\frac{m_0c^2-E}{\hbar c}\right)\right)$$

$$\to \cancel{2}\,\frac{\sqrt{(m_0^2c^4 - E^2)}}{\cancel{\hbar c}}\underbrace{(n+q)}_{N'} = Z\alpha_f\left(\frac{\cancel{2}E}{\cancel{\hbar c}}\right)$$

$$(m_0^2 c^4 - E^2) N'^2 = Z^2 \alpha_f^2 E^2$$

$$E^2 \left((Z\alpha_f)^2 + N'^2 \right) = m_0^2 c^4 N'^2$$

$$E^2 = \frac{m_0^2 c^4 N'^2}{(Z\alpha_f)^2 + N'^2} = \frac{m_0^2 c^4}{1 + \frac{(Z\alpha_f)^2}{N'^2}}$$

$$E = m_0 c^2 \cdot \left(1 + \frac{(Z\alpha_f)^2}{N'^2} \right)^{-\frac{1}{2}} \checkmark$$

$$\alpha \varepsilon_+ \varepsilon_- \cancel{\nu} = \frac{\cancel{\nu}}{2b}$$

$$// \; n = \emptyset$$

$$\rightarrow \quad b = \frac{1}{2\varepsilon_+ \varepsilon_-} \qquad 2\varepsilon_+\varepsilon_- = \frac{Z\alpha_f (\varepsilon_+^2 - \varepsilon_-^2)}{q}$$

$$= \frac{2E \, Z\alpha_f}{q}$$

$$\rightarrow \quad b = \frac{q}{2E \, Z\alpha_f} = \frac{\sqrt{(j+\frac{1}{2})^2 - (Z\alpha_f)^2}}{2E \; Z\alpha_f}$$

$$N = n + j + \frac{1}{2}$$

$$= \frac{\sqrt{N^2 - (Z\alpha)^2}}{2E \; Z\alpha}$$

$$\frac{\sqrt{N^2 - (Z\alpha)^2}}{(Z\alpha)} = \sqrt{\frac{N^2 - Z^2}{Z^2}} = \sqrt{\frac{N^2}{Z^2} - 1}$$

Moleküle :

$$c_a A + c_B B = \varnothing$$

$$c_b A + c_A B = \varnothing$$

$$c_b = -\frac{c_A B}{A}$$

$$c_a A - \frac{c_A B^2}{A} = \varnothing$$

$$c_a \left(A - \frac{B^2}{A} \right) = \varnothing \qquad // \cdot A$$

$$c_a (A^2 - B^2) = \varnothing \qquad \checkmark$$