Edyn S30

Energiesatz:

$$\operatorname{rot}\vec{E} + \frac{1}{c}\partial_t \vec{B} = \emptyset \qquad / \cdot \vec{B}$$
$$\operatorname{rot}\vec{B} - \frac{1}{c}\partial_t \vec{E} = \frac{4\pi}{c}\vec{j} \qquad / \cdot \vec{E}$$

$$\left.\begin{array}{l} (\operatorname{rot}\vec{E})\vec{B} + \left(\frac{1}{c}\partial_t \vec{B}\right)\vec{B} = \emptyset \qquad / \cdot \frac{c}{4\pi} \\ (\operatorname{rot}\vec{B}) E - \left(\frac{1}{c}\partial_t \vec{E}\right)\vec{E} = \frac{4\pi}{c}\vec{j}\vec{E} \qquad / \cdot \frac{c}{4\pi} \end{array}\right\} -$$

$$-\frac{c}{4\pi}\underbrace{\left(\vec{B}\operatorname{rot}\vec{E} - \vec{E}\operatorname{rot}\vec{B}\right)}_{NR2} - \frac{1}{4\pi}\underbrace{\left(\vec{B}\dot{\vec{B}} + \vec{E}\dot{\vec{E}}\right)}_{NR1} = \vec{j}\vec{E}$$

NR1:

$$\partial_t(\vec{B}^2) = 2\vec{B}\dot{\vec{B}} \Rightarrow -\frac{1}{8\pi}\partial_t\left(\vec{E}^2 + \vec{B}^2\right)$$

NR2:

$$B_i \varepsilon_{ijk}\partial_j E_k \underbrace{- E_i \varepsilon_{ijk}\partial_j B_k}_{+ E_k \varepsilon_{ijk}\partial_j B_i}$$

$$\varepsilon_{ijk}\partial_j(B_i E_k) = \varepsilon_{ijk}(B_i \partial_j E_k + E_k \partial_j B_i)$$

$$\Rightarrow \varepsilon_{ijk}\partial_j(B_i E_k) = \partial_j \varepsilon_{jki} E_k B_i$$
$$= \operatorname{div}(\vec{E}\times\vec{B})$$

$$\Rightarrow \vec{j}\vec{E} = -\frac{c}{4\pi}\operatorname{div}(\vec{E}\times\vec{B}) - \frac{1}{8\pi}\partial_t\left(\vec{E}^2 + \vec{B}^2\right)$$

$$\vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{B}) \qquad w_{em} = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{B}^2\right)$$

$$\frac{d A_{mech}}{dt} = \int d^3r\, \vec{j}\vec{E}$$
$$= \int d^3r\, -\operatorname{div}\vec{S} - \partial_t w_{em}$$

$$\int d^3r\, w_{em} = A_{feld}$$

$$\Rightarrow \frac{d}{dt}(A_{mech} + A_{Feld}) = -\int_V d^3r\, \operatorname{div}\vec{S} = \int_F d^2\vec{f}\,\vec{S}$$

// Man braucht dafür nur die 2 rot Gleichungen!

S 31 Impulssatz

$$\vec{F} = \frac{d}{dt}\vec{P}_{mech} = \int d^3r\,\left(\rho\vec{E} + \rho\frac{\vec{v}}{c}\times\vec{B}\right) = \int d^3r\,\rho\vec{E} + \frac{\vec{j}}{c}\times\vec{B}$$

Man braucht alle MWGs dafür!

$$\left.\begin{aligned} \operatorname{rot}\vec{E} + \frac{1}{c}\partial_t\vec{B} &= \emptyset \quad &/\times\vec{E} \\ \operatorname{rot}\vec{B} - \frac{1}{c}\partial_t\vec{E} &= \frac{4\pi}{c}\vec{j} \quad &/\times\vec{B} \end{aligned}\right]_{1)}^{+}$$

$$\left.\begin{aligned} \operatorname{div}\vec{E} &= 4\pi\rho \quad &/\cdot\vec{E} \\ \operatorname{div}\vec{B} &= \emptyset \quad &/\cdot\vec{B} \end{aligned}\right]_{2)}^{+}$$

beide für d 4π div!

2) $\vec{E}\operatorname{div}\vec{E} + \vec{B}\operatorname{div}\vec{B} = 4\pi\rho\vec{E}$

1) $\underbrace{\operatorname{rot}\vec{E}\times\vec{E} + \operatorname{rot}\vec{B}\times\vec{B}}_{NR2} + \frac{1}{c}\underbrace{(\dot{\vec{B}}\times\vec{E} - \dot{\vec{E}}\times\vec{B})}_{NR1} = \frac{4\pi}{c}\vec{j}\times\vec{B}$

NR 1) $(\partial_t\vec{B})\times\vec{E} - (\partial_t\vec{E})\times\vec{B} = \dot{\vec{B}}\times\vec{E} + B\times\dot{\vec{E}} =$

$-\partial_t(\vec{E}\times\vec{B})$

NR 2) $\varepsilon_{ijk}(\partial_j E_k)\underbrace{\varepsilon_{pil}}_{\varepsilon_{ilp}}E_l = (\delta_{jl}\delta_{kp} - \delta_{jp}\delta_{kl})(\partial_j E_k)E_l$

$= (\partial_l E_p)E_l - (\partial_p E_l)E_l$

2) + 1) : $\underbrace{E_i\partial_k E_k + (\partial_k E_i)E_k}_{\partial_k(E_iE_k)} - (\partial_i E_k)E_k$

$\partial_k(E_iE_k) - \frac{1}{2}\underbrace{\partial_i(E_kE_k)}_{\partial_k\delta_{ik}(E_lE_l)}$

$\Rightarrow \frac{1}{4\pi}\partial_k\left(E_iE_k + B_iB_k - \frac{1}{2}\delta_{ik}(E_lE_l + B_lB_l)\right) - \frac{1}{4\pi c}\partial_t(\varepsilon_{klm}E_lB_m)$

$= \rho E_k + \frac{1}{c}\varepsilon_{klm}j_lB_m$

$T_{ik} = \frac{1}{4\pi}\left(E_iE_k + B_iB_k - \frac{1}{2}\delta_{ik}(E_lE_l + B_lB_l)\right)$

$g_{em} = \frac{1}{4\pi c}(\vec{E}\times\vec{B}) \qquad \vec{P}_{feld}(t) = \int_V d^3r\, g_{em}$

$$\Rightarrow \quad \frac{d}{dt}\left(\vec{P}_{mech} + \vec{P}_{feld}\right) = \int_F d^2\vec{f}\ \overleftrightarrow{T}$$

§38 Entwicklung von $\frac{1}{|\vec{r}-\vec{r}\,'|} = \left(x_i x_i - 2x_i x_i' + x_i' x_i'\right)^{-\frac{1}{2}}$

$$f(x_k) = \underbrace{f(a_k)}_{1.T} + \underbrace{\partial_i f(a_k)(x_i - a_i)}_{2.T} + \underbrace{\frac{1}{2}\partial_i\partial_j f(a_k)(x_i - a_i)(x_j - a_j)}_{3.T} + \dots$$

Entwickeln um $r' = \emptyset$

1) $\frac{1}{r}$

2) $\partial_i' f(a_k) := \partial_i'\left(x_k x_k - 2x_k x_k' + x_k' x_k'\right)^{-\frac{1}{2}}$

$$= -\frac{1}{\cancel{2}}\left(\quad\right)^{-\frac{3}{2}}\left(-\cancel{2}x_i + \cancel{2}x_i'\right) = \frac{1}{|\vec{r}-\vec{r}\,'|^3}\left(\vec{r}-\vec{r}\,'\right)$$

$$\partial_i' f(\vec{r}\,' = \vec{0}) = \frac{\vec{r}}{r^3} = A \qquad A\cdot\left(\vec{r}\,' - \vec{\emptyset}\right) = \frac{\vec{r}\,\vec{r}\,'}{r^3}$$

$$\boxed{\begin{aligned} &\partial_j' x_i' = \delta_{ji} \\ &\partial_j'(x_i x_i') = x_i\,\delta_{ji} = x_j \\ &\partial_j'(x_i' x_i') = 2x_i\,\delta_{ji} = 2x_j \\ &\Rightarrow \vec{\nabla}(\vec{x}^2) = 2\vec{x} \end{aligned}}$$

3) $\partial_i'\partial_j' f(a_k) = \partial_i'\left(\quad\right)^{-\frac{3}{2}}(x_j - x_j')$

$$= 3\left(\quad\right)^{-\frac{5}{2}}(x_i - x_i')(x_j - x_j')$$

$$+ \left(\quad\right)^{-\frac{3}{2}}(-\delta_{ij})$$

$$= \left.\frac{3(r_i - r_i')(r_j - r_j') - |\vec{r}-\vec{r}\,'|^2\,\delta_{ij}}{|\vec{r}-\vec{r}\,'|^5}\right|_{\vec{r}\,'=\vec{0}}$$

$$= \frac{3\,r_i\,r_j - r^2\delta_{ij}}{r^5} \Rightarrow \frac{1}{2}\,\frac{(3\,r_i r_j - r^2\delta_{ij})}{r^5}\,r_i' r_j'$$

$$= \frac{1}{2}\,\frac{3\,r_i r_i' r_j r_j' - r^2 r_i' r_j'\,\delta_{ij}}{r^5}$$

$$= \frac{1}{2}\,\frac{3(\vec{r}\,\vec{r}\,')^2 - r^2 r'^2}{r^5}$$

§39 Dipolfeld

$$\vec{p} = \int d^3r'\ \vec{r}\,'\,\rho(\vec{r}\,')$$

$$\phi_{Dipol} = \frac{\vec{r}\,\vec{p}}{r^3} \longrightarrow E_i = -\partial_i\,\phi = \partial_i\frac{r_k p_k}{r_l r_l^{\frac{3}{2}}}$$

$$= -\partial_i(r_k p_k)\frac{1}{r^3} - \partial_i\left(r_l r_l\right)^{-\frac{3}{2}} r_k p_k$$

$$= -\frac{p_i}{r^3} + \frac{3}{\cancel{2}} \cancel{2} r_i \, r_k p_k \cdot (r_l r_l)^{-\frac{5}{2}} = \frac{-p_i r^2 + 3 r_i r_k p_k}{r^5} \checkmark$$

S4.1) MW d. E-Feldes im Kugelbereich $r < R$

$$\langle E \rangle = \frac{1}{V} \int_{r<R} \vec{E}(\vec{r}) \, d^3 r = \frac{3}{4 R^3 \pi} \int_{r<R} -\operatorname{grad} \phi$$

$$\boxed{\phi(\vec{r}) = \int d^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}} = -\frac{3}{4 R^3 \pi} \int_{r<R} d^3 r \, \partial_i \int d^3 r' \frac{\rho(r')}{|\vec{r} - \vec{r}'|}$$

$$= -\frac{3}{4 R^3 \pi} \oint_{r=R} d^2 \vec{f} \int d^3 \vec{r}' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$= -\frac{3}{4 R^3 \pi} \int d^3 r' \rho(\vec{r}') \int_{r=R} d^2 \vec{f} \frac{1}{|\vec{r} - \vec{r}'|} \qquad \vec{r}' = r' \cdot \vec{e}_z$$

$$\vec{I} = \oint_{r=R} d^2 \vec{f} \frac{1}{|\vec{r} - \vec{r}'|} \qquad d^2 \vec{f} = d^2 f \, \vec{e}_r$$

$$\vec{I}(\vec{r}') = \oint_{r=R} d^2 \vec{f} \frac{1}{|\vec{r} - \vec{r}'|} \qquad \vec{I} = I(r', R) \cdot \vec{e}_{r'} \qquad \vec{e}_{r'} = \vec{e}_z$$

$$\vec{I} \cdot \vec{e}_{r'} = I$$

$$\Rightarrow I = R^2 \int d\Omega \frac{\vec{e}_r \cdot \vec{e}_{r'}}{|\vec{r} - \vec{r}'|} \qquad \vec{e}_r \cdot \vec{e}_{r'} = \vec{e}_r \cdot \vec{e}_z = \cos\vartheta$$

$$= R^2 \int d\Omega \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{r_<^l}{r_>^{l+1}} Y^*_{lm}(\vartheta_<, \varphi_<) \, Y_{lm}(\vartheta_>, \varphi_>) \cdot \cos\vartheta$$

$\vartheta' = \emptyset$, weil z-Achse

$r' > R$: $\frac{4\pi}{2l+1}$ $\qquad // \; Y_{l,-m} = (-1)^m Y^*_{lm}$

$$I = R^2 \int d\Omega \sum_l \sum_m \frac{R^l}{r'^{l+1}} Y^*_{lm}(\vartheta, \varphi) \, Y_{lm}(\vartheta', \varphi') \cdot \sqrt{\frac{4\pi}{3}} Y^*_{10}(\vartheta, \varphi)$$

$$\sqrt{\frac{4\pi}{3}} \qquad \delta_{l1} \delta_{m0}$$

$$\int d\Omega \, Y^*_{l'm'}(\vartheta, \varphi) \, Y_{lm}(\vartheta, \varphi) = \delta_{ll'} \delta_{mm'}$$

$$I = R^2 \sum_{\ell m} \frac{R^\ell}{r'^{\ell+1}} Y_{\ell m}(\vartheta' = \emptyset) \sqrt{\frac{4\pi}{3}} \delta_{\ell 1} \delta_{m 0} \cdot \frac{4\pi}{2\ell+1}$$

$$= R^2 \frac{R^1}{r'^2} \underbrace{Y_{10}(\vartheta' = \emptyset) \sqrt{\frac{4\pi}{3}}}_{\cos(\vartheta' = 0) = 1} \cdot \frac{4\pi}{3} = \frac{R^3}{r'^2}$$

$$\frac{1}{V} \cdot I = \frac{\cancel{3}}{\cancel{4\pi} \cancel{R^3}} \cdot \frac{\cancel{R^3} \cancel{4\pi}}{r'^2 \cancel{3}} = \underline{\frac{1}{r'^2}} \checkmark$$

$r' < R$:

$$I = R^2 \int d\Omega \sum_{\ell m} \frac{4\pi}{2\ell+1} \frac{r'^\ell}{R^{\ell+1}} Y^*_{\ell m}(\vartheta', \varphi') \underbrace{Y_{\ell m}(\vartheta, \varphi) \sqrt{\frac{4\pi}{3}} Y_{10}(\vartheta, \varphi)}_{\delta_{1\ell} \delta_{0m}}$$

$$= \cancel{R^2} \frac{4\pi}{3} \frac{r'^1}{\cancel{R^2}} \underbrace{Y^*_{10}(\vartheta' = 0) \sqrt{\frac{4\pi}{3}}}_{\cos(\vartheta' = 0) = 1} = \frac{4\pi}{3} r'$$

$$\frac{1}{V} I = \frac{\cancel{3}}{\cancel{4\pi} R^3} \frac{\cancel{4\pi} r'}{\cancel{3}} = \underline{\frac{r'}{R^3}}$$

S42 E - statische Energie

$$W = \frac{1}{8\pi} \int_V d^3r \, \vec{E}^2(\vec{r}) \qquad \vec{E} = -\vec{\nabla} \phi(\vec{r})$$

$$\underbrace{(\nabla \phi)}_{a} \underbrace{(\nabla \phi)}_{\nabla b} \qquad \boxed{a \nabla b = \nabla a b - b \nabla a}$$

$$\Delta \phi = -4\pi \rho$$

$$= \nabla (\nabla \phi) \phi - \phi \Delta \phi$$

$$\Rightarrow W = \frac{1}{8\pi} \Big(\underbrace{\int_F d^2\vec{f} \, \phi \nabla \phi}_{\text{fällt weg bei nat. RB}} + 4\pi \int_V \phi \rho \, dV \Big)$$

Kapazitätskoeff ... S 43

$$\phi = \int_V G \rho \, dV + \frac{1}{4\pi} \oint_F \left(G \nabla' \phi - \phi \nabla' G \right)$$

Geg: n Leiter tragen je Q_i Ladung (auf Oberflächen)

$\longrightarrow$ 1. Term fällt weg   Dirichlet RB (Potential bekannt auf F)

$\longrightarrow$ G_D auf $F = \varnothing$

$$\Longrightarrow \underbrace{\phi}_{\phi(\vec{r})} = \overbrace{+\frac{1}{4\pi}}^{*_1} \sum_n \oint_{F_n} \phi \nabla' G_D \, d^2\vec{f}'$$

Vol ist eig. ganzer Raum mit Rändern ... $\vec{d}^2 f_n$ zeigt in die n. Kugel

In der Formel zeigt jed. $\vec{d}^2 f$ aus dem Leiter $\rightarrow$ VZ-Wechsel $*_1$

L-Oberfläche $\quad \vec{E} = 4\pi\sigma \vec{n}$

$$Q_m = \oint_{F_m} \frac{1}{4\pi} \underbrace{\vec{E}}_{-\vec{\nabla}\phi} \, d^2\vec{f}$$

$$= \sum_n \underbrace{-\frac{1}{(4\pi)^2} \oint_{F_m} d^2\vec{f} \, \vec{\nabla} \oint_{F_n} d^2\vec{f}' \, \nabla' G_D}_{C_{mn}} \phi_n \qquad Q_m = \sum_n C_{mn} \phi_n$$

$$W = \frac{1}{2} \int_V \rho \phi \, d^3 r = \frac{1}{2} \sum_{m,n} C_{mn} \phi_n \phi_m \qquad // \text{nat. R.B}$$

C zweier Leiter S 44

$$Q = CU \longrightarrow C = \frac{Q}{\phi_1 - \phi_2}$$

$$Q_1 = -Q_2 = Q \qquad Q_1 = C_{11}\phi_1 + C_{12}\phi_2$$
$$Q_2 = C_{21}\phi_1 + C_{22}\phi_2$$

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

Lösen d. lin Gl. Sys.

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{\det C_{mn}} \begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$$

Lokale Ladungsvert. in äußerem Feld § 44

$$W^{ww}(\vec{R}) = \int_V \phi(\vec{r})\, \rho(\vec{r} - \vec{R})\, d^3r \quad // \text{ verschieben} \quad \tilde{\vec{r}} = \vec{r} - \vec{R}$$

$$\rightarrow \int_V \phi(\vec{r} + \vec{R})\, \rho(\vec{r})\, d^3r$$

Entwickeln v. $\phi(\vec{r} + \vec{R})$ um $\vec{R}$

$$\vec{x}_0 := R$$

$$f(\vec{x}) = f(\vec{x}_0) + \partial_i f(\vec{x}_0)(\vec{x} - \vec{x}_0) + \frac{1}{2} \partial_i \partial_j f(\vec{x}_0)(\vec{x} - \vec{x}_0)_i (\vec{x} - \vec{x}_0)_j$$

$$\Rightarrow \phi(\underbrace{\vec{R} + \vec{r}}_{\vec{x}}) = \phi(\vec{R}) + \nabla_R \phi(\vec{R})(\cancel{\vec{R}} + \vec{r} - \cancel{\vec{R}}) + \frac{1}{2} \frac{\partial^2 \phi}{\partial R_i \partial R_j} x_i x_j$$

$$\rightarrow W^{ww} = \phi(\vec{R})\, q + \nabla_R \phi(\vec{R}) \underbrace{\int_V \vec{r} \rho\, d^3r}_{\vec{p}} + \frac{1}{2} \overbrace{\frac{\partial^2 \phi}{\partial R_i \partial R_j}}^{-\frac{\partial E_i}{\partial R_j}} \cdot \int x_i x_j \rho\, d^3r$$

$$= q\, \phi(\vec{R}) - \vec{p}\, \vec{E}(\vec{R}) - \frac{1}{2} \frac{\partial E_i(\vec{R})}{\partial R_j} \underbrace{\int x_i x_j \rho\, d^3r}_{\text{Umschr. in } Q = 3x_i x_j - \delta_{ij} r^2}$$

Term 2. ist $\frac{\partial E_i}{\partial R_j} \delta_{ij} = \frac{\partial E_i}{\partial R_i} = \emptyset$ innerhalb

$$= q\, \phi - \vec{p}\, \vec{E} - \frac{1}{6} \frac{\partial E_i}{\partial R_j} Q_{ij}$$

Kräfte in E-Feldern § 46

// LV. um $\vec{R}$

$$F = \int_V \rho(\vec{r} - \vec{R})\, \vec{E}(\vec{r}) \qquad \tilde{\vec{r}} = \vec{r} - \vec{R}$$

$$= \int_V \rho(\vec{r})\, \vec{E}(\vec{R} + \vec{r}) \qquad // \text{ Entw. v. } \vec{E}$$

$$\vec{E}(\vec{R} + \vec{r}) = \vec{E}(\vec{R}) + (\vec{\nabla}_R \vec{E})\vec{r} + \frac{1}{2} \frac{\partial^2 \vec{E}}{\partial R_i \partial R_j} x_i x_j \qquad // \text{ analog zu } \phi$$

$$\Rightarrow F = q\vec{E}(\vec{R}) + (\vec{\nabla}_R \vec{E}(\vec{R})) \underbrace{\int \rho \vec{r}}_{\vec{p}} + \dots$$

$$\epsilon_{ijk}\, p_j\, \epsilon_{klm}\, \partial_l E_m = \epsilon_{ijk}\epsilon_{lmk} \text{ -''- } = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) \text{ -''- }$$

$$= p_m \partial_i E_m - p_l \partial_l E_m$$

$$\Rightarrow \vec{p} \times \underbrace{(\vec{\nabla} \times \vec{E})}_{0} = \vec{\nabla}(\vec{p}\vec{E}) - (\vec{p}\vec{\nabla})\vec{E}$$

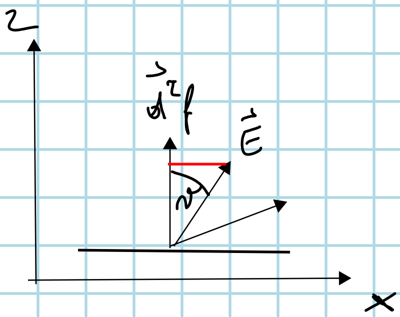
herausziehen, da $p = \text{const}$

$$\Rightarrow \vec{F} = q\vec{E} + \vec{\nabla}(\vec{p}\vec{E}) = -\vec{\nabla} W^{ww}$$

§ 47 Kraft auf ein Vol. ausgehend vom Spannungstensor

$$T_{ij} = \frac{1}{4\pi}(E_i E_j + B_i B_j) - \frac{1}{2}\delta_{ij}(\vec{E}^2 + \vec{B}^2)$$

e-statisch: $T_{ij} = \frac{1}{4\pi} E_i E_j - \frac{1}{2}\delta_{ij}\vec{E}^2$



zuerst ϑ festlegen, da E in $x-z$ Ebene

$$\rightarrow \vec{E} = E\begin{pmatrix} \sin\vartheta \\ 0 \\ \cos\vartheta \end{pmatrix}$$

Flächenkraft $\vec{T} = \vec{n}\,\overleftrightarrow{T} \Rightarrow T_i = e_{z,j} T_{ij}$ $\quad \vec{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$T = \begin{pmatrix} T_{13} \\ T_{23} \\ T_{33} \end{pmatrix} \qquad T_{ij} = \begin{pmatrix} E_x^2 - \frac{\vec{E}^2}{2} & E_x E_y & E_x E_z \\ E_y E_x & E_y^2 - \frac{\vec{E}^2}{2} & E_y E_z \\ E_z E_x & E_z E_y & E_z^2 - \frac{\vec{E}^2}{2} \end{pmatrix}$$

$$\Rightarrow T = \begin{pmatrix} E_x E_z \\ E_y E_z \\ E_z^2 - \frac{\vec{E}^2}{2} \end{pmatrix} = \begin{pmatrix} E^2 \cos\vartheta \sin\vartheta \\ 0 \\ E^2\left(\cos^2\vartheta - \frac{1}{2}\underbrace{(\cos^2\vartheta + \sin^2\vartheta)}_{1}\right) \end{pmatrix}$$

$$T = \frac{E^2}{4\pi}\begin{pmatrix} \frac{1}{2}\, 2\cos\vartheta \sin\vartheta \\ \emptyset \\ \frac{1}{2}(2\cos^2\vartheta - 1) \end{pmatrix} = \frac{E^2}{8\pi}\begin{pmatrix} \sin 2\vartheta \\ \emptyset \\ \cos 2\vartheta \end{pmatrix}$$

§ 49 Magnetostatik im Vakuum

$$\vec{A}(\vec{r}) = \frac{1}{c}\int \frac{\vec{j}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\, d^3r'$$

$$\vec{B} = \text{rot}\,\vec{A} : \quad \varepsilon_{ijk}\partial_j \left((\vec{r} - \vec{r}\,')^2\right)^{-\frac{1}{2}}$$

$$= \varepsilon_{ijk}\partial_j \left(r^2 - 2\vec{r}\vec{r}\,' + r'^2\right)^{-\frac{1}{2}}$$

$$= \varepsilon_{ijk}\; -\frac{1}{\cancel{2}}\left(\;-\;\text{''}\;-\;\right)^{-\frac{3}{2}}(\cancel{2}\vec{r} - \cancel{2}\vec{r}\,')_j$$

$$// \quad \partial_j r^2 = \partial_j(x_i x_i) = 2x_j$$

$$\vec{\nabla}\times\vec{A} = -\frac{1}{c}\int \frac{(\vec{r} - \vec{r}\,')\times \vec{j}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}\, d^3r'$$

$$\Rightarrow \vec{B} = \frac{1}{c}\int_V \frac{\vec{j}\times(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$$

§ 50 Magn. Dipolmoment

$$\frac{1}{|\vec{r} - \vec{r}\,'|} = \frac{1}{r} + \frac{\vec{r}\,\vec{r}\,'}{r^3} + \frac{1}{2}\,\frac{3(\vec{r}\,\vec{r}\,')^2 - r^2 r'^2}{r^5}$$

einsetzen in $\vec{A}(\vec{r})$

§ 57 Magnetostatische Energie

$$\vec{B}^2 = \vec{B}\cdot(\vec{\nabla}\times\vec{A})$$

$$B_i\,\varepsilon_{ijk}\partial_j A_k \implies \varepsilon_{ijk}\partial_j(B_i A_k) - \varepsilon_{ijk} A_k \partial_j B_i$$

$((\vec{\nabla}\times\vec{A})\vec{B})$ $\qquad$ $(\vec{A}\times\vec{\nabla})\vec{B}$

$$// \quad \partial_j(B_i A_k) = A_k\partial_j B_i + B_i\partial_j A_k$$

$$= \varepsilon_{jki}\partial_j A_k B_i + \varepsilon_{kji} A_k \partial_j B_i$$

$\vec{\nabla}(\vec{A}\times\vec{B})$ $\qquad$ $\vec{A}\cdot(\vec{\nabla}\times\vec{B})$

S 60 Kräfte in magn. Feldern

$$\varepsilon_{ijk}\, a_j\, \varepsilon_{klm}\, b_l\, c_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\, a_j\, b_l\, c_m$$
$$= a_m\, b_i\, c_m - a_l\, b_l\, c_i$$

aber hier zu verwenden: $\varepsilon_{ijk}\, m_j\, \partial_k\, \varepsilon_{lim}\, B_m$

$$\varepsilon_{ijk}\,\varepsilon_{lim} = \varepsilon_{ijk}\,\varepsilon_{iml} = \delta_{jk}\delta_{ml} - \delta_{jl}\delta_{km} \;//\; m_j\, \partial_k B_m$$

$$\Rightarrow \underline{\underline{m_k\, \partial_k B_l}} - m_l\, \underbrace{\partial_m B_m}_{0}$$

$$\partial_k (m_k B_l) = B_l \underbrace{\partial_k m_k}_{0} + m_k \partial_k B_l \quad ✓$$

S 65 period. ebene Wellen

$$\vec{E} = \vec{E}_0\, f(\vec{n}\vec{r} - ct) \qquad \text{rot}\, \vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}$$
$$(\vec{n} \times \vec{E}_0) f' = -\frac{1}{c}\partial_t \vec{B}$$

Ansatz: $\vec{B} = (\vec{n} \times E_0)\, f(\vec{n}\vec{r} - ct)$
$$-\frac{1}{c}\partial_t \vec{B} = -\frac{1}{c} f' \cdot (-c)\; (\vec{n} \times \vec{E}_0) = (\vec{n} \times \vec{E}_0) f'$$

oder integrieren? $\int (\vec{n} \times \vec{E}_0) f'\, dt = \int -\frac{1}{c}\partial_t \vec{B}\, dt$
$$(\vec{n} \times \vec{E}_0) f \cdot \frac{1}{\cancel{-c}} = -\frac{1}{\cancel{c}}\vec{B} \quad ✓$$

innere Ableitung v. $\int$

S 85

$$\frac{1}{c}\partial_t E_i = \varepsilon_{ijk}\,\partial_j B_k$$

$$= \varepsilon_{ijk}\,\partial_j\,\varepsilon_{klm}\,\ddot{q}_l\,r_m \cdot \frac{1}{c^2 r^2}$$

$$= \frac{1}{c^2}\,\varepsilon\,\varepsilon\left(\ddot{q}_l\,r_m\,\partial_j\left(\frac{1}{r^2}\right) + \frac{1}{r^2}\,\partial_j\left(\ddot{q}_l\,r_m\right)\right)$$

$\to \frac{1}{r}$ viel

$$= \frac{1}{c^2}\,\varepsilon\varepsilon\,\underbrace{\frac{1}{r^2}\left(\ddot{q}_l\,\delta_{jm}\right.}_{\frac{1}{r^2}\;\times} \left. + r_m\,\partial_j\,\ddot{q}_l\right)$$

$$= \frac{1}{c^2}\,\varepsilon\,\varepsilon\,\frac{1}{r^2}\,r_m\,\partial_j\,\ddot{q}_l$$

$$\partial_j\,\ddot{q}_l = \dddot{q}_l\,\partial_j\left(t - \frac{|r-r'|}{c}\right) = -\,\dddot{q}_l\,\frac{(r-r')_j}{|r-r'|}$$

$r' = \emptyset$ weg Reihenentw.!

$$= \frac{1}{c^2}\,\varepsilon\varepsilon\,\frac{1}{r^2}\,r_m\,-\,\dddot{q}_l\,\frac{r_j}{r}$$

$$= -\frac{1}{c^2 r^3}\,\varepsilon_{ijk}\,\varepsilon_{klm}\,r_j\,\ddot{q}_l\,r_m = \frac{1}{c^2 r^3}\,\vec{\ddot{q}}\times\vec{r}\times\vec{r}$$

✓

S 168 44 a $\rightarrow$ 45 a

$$\Re\,\varepsilon(\omega) = 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\Im\,\varepsilon(\omega')}{\omega' - \omega}$$

mit $\Im\,\varepsilon(-\omega) = -\Im\,\varepsilon(\omega)$

$$\Re\,\varepsilon(\omega) = 1 + \frac{1}{\pi} P \left(\int_{-\infty}^{0} d\omega' \frac{\Im\,\varepsilon(\omega')}{\omega' - \omega} + \int_{0}^{\infty} \text{—''—} \right)$$

$$= 1 + \frac{1}{\pi} P \left(\underbrace{\int_{-\infty}^{0} d\omega' \frac{-\Im\,\varepsilon(-\omega')}{\omega' - \omega}} + \text{—''—} \right)$$

$\tilde{\omega}' = -\omega'$

$\frac{d\tilde{\omega}'}{d\omega'} = -1$

$$\underbrace{\int_{\infty}^{0} d\tilde{\omega}' \frac{\Im\,\varepsilon(\tilde{\omega}')}{-\tilde{\omega}' - \omega}}$$

// Grenzen umtauschen

$$\rightarrow \int_0^{\infty} d\omega' \frac{\Im\,\varepsilon(\omega')}{\omega' + \omega}$$

$$\Rightarrow \Re\,\varepsilon(\omega) = 1 + \frac{1}{\pi} P \left(\int_0^{\infty} d\omega' \underbrace{\frac{\Im\,\varepsilon(\omega')}{\omega' + \omega} + \frac{\Im\,\varepsilon(\omega')}{\omega' - \omega}}_{\text{gl. Nenner}} \right)$$

$$\int_0^{\infty} d\omega' \frac{\Im\,\varepsilon(\omega')(\omega' - \omega) + \Im\,\varepsilon(\omega')(\omega' + \omega)}{\omega'^2 - \omega^2} = \int \frac{\Im\,\varepsilon(\omega')(\omega' \cancel{-\omega} + \omega' \cancel{+\omega})}{\text{—''—}}$$

$$\int d\omega' \frac{2\omega' \Im\,\varepsilon(\omega')}{\omega'^2 - \omega^2} \checkmark$$