Potentiale & Eichungen

homogene MWG

$\partial_i B_i = 0 \quad \longrightarrow \quad B_i = \varepsilon_{ijk}\partial_j A_k$

$\mathrm{rot}\, E + \frac{1}{c}\partial_t B = 0$

$\mathrm{rot}\, E + \frac{1}{c}\partial_t \,\mathrm{rot}\, A = 0$

$\mathrm{rot}\,(E + \frac{1}{c}\partial_t A) = 0$

$\longrightarrow \quad E_i + \frac{1}{c}\partial_t A_i = -\partial_i \phi$

$E_i = -\partial_i \phi - \frac{1}{c}\partial_t A_i$

jetzt inhomogene

$\partial_i E_i = 4\pi\rho$

$\mathrm{rot}\, B - \frac{1}{c}\partial_t E = \frac{4\pi}{c} j$

1) $\partial_i\left(-\partial_i \phi - \frac{1}{c}\partial_t A_i\right) = 4\pi\rho$

2) $\underbrace{\mathrm{rot}\,\mathrm{rot}\, A_k}_{\mathrm{grad}\,\mathrm{div}\, A_k - \Delta A_i} - \frac{1}{c}\partial_t\left(-\partial_i\phi - \frac{1}{c}\partial_t A_i\right) = \frac{4\pi}{c} j_i$

$/\cdot -1$

$\frac{1}{c^2}\partial_t^2 A_i - \Delta A_i + \partial_i\left(\partial_k A_k + \frac{1}{c}\partial_t \phi\right) = \frac{4\pi}{c} j_i$

$\Box A_i - \partial_i\left(\partial_k A_k + \frac{1}{c}\partial_t\phi\right) = -\frac{4\pi}{c} j_i$ ✓

1 wieder) $\quad -\Delta\phi + \frac{1}{c^2}\partial_t^2\phi - \frac{1}{c}\partial_t\left(\frac{1}{c}\partial_t\phi + \partial_k A_k\right) = 4\pi\rho$

$/\cdot -1$

$\Box\phi + \frac{1}{c}\partial_t\left(\partial_k A_k + \frac{1}{c}\partial_t\phi\right) = -4\pi\rho$

Lorenz Eichung $\quad \partial_k A_k^L + \frac{1}{c}\partial_t \phi^L = \emptyset$

$$\rightarrow \quad \Box A_i^L = -\frac{4\pi}{c} j_i$$

$$\Box \phi_i^L = -4\pi\rho$$

Eichtrafo : $\quad B = \mathrm{rot}\, A$

$$\rightarrow \quad \varepsilon_{ijk}\, \partial_j \partial_k \psi = \emptyset \qquad \underline{A_k' = A_k - \partial_k \psi}$$

$$E_i = -\partial_i \phi' - \frac{1}{c}\partial_t A_i'$$

// was setzt man ein, damit sich das ψ weghebt?

$$\underline{\phi' = \phi + \frac{1}{c}\partial_t \psi}$$

$$E = -\partial_i(\phi + \partial_t\psi) - \frac{1}{c}\partial_t(A_i - \partial_i \psi)$$

✓

$$\partial_k(A_k - \partial_k\psi) + \frac{1}{c}\partial_t\left(\phi + \frac{1}{c}\partial_t\psi\right) = \emptyset$$

$$-\partial_k\partial_k\psi + \partial_k A_k + \frac{1}{c^2}\partial_t^2\psi + \frac{1}{c}\partial_t\phi = \emptyset$$

$$\Box\psi = \partial_k A_k + \frac{1}{c}\partial_t\phi$$

Uneindeutig, wenn $\Box(\psi + \psi') = $ — " —

$$\Box\psi + \underbrace{\Box\psi'}_{\emptyset} = \mathrm{div}\ ---$$

✓

Coulomb - Eichung

1) $\Box A_i - \partial_i\left(\partial_k A_k + \frac{1}{c}\partial_t\phi\right) = -\frac{4\pi}{c} j_i$

2) $\Box\phi + \frac{1}{c}\partial_t\left(\partial_k A_k + \frac{1}{c}\partial_t\phi\right) = -4\pi\rho$

$\partial_k A_k^T = \emptyset$ transversal

2) $\Delta \phi \cancel{- \frac{1}{c^2}\partial_t^2 \phi} + \cancel{\frac{1}{c^2}\partial_t^2 \phi} + \frac{1}{c}\partial_t \underbrace{\partial_k A_k}_{\emptyset} = -4\pi\rho$

$\rightarrow \Delta \phi = -4\pi\rho$

1) $\partial_j \partial_j A_i^T - \frac{1}{c^2}\partial_t^2 A_i^T - \partial_i \underbrace{\partial_k A_k}_{\emptyset} - \frac{1}{c}\partial_t \partial_i \phi = -\frac{4\pi}{c} j_i$

$$\Box \vec{A}^T = -\frac{4\pi}{c} j_i + \frac{1}{c}\partial_t \partial_i \phi_c$$

$$= -\frac{4\pi}{c}\underbrace{\left(j_i - \frac{1}{4\pi}\partial_i \partial_t \phi_c\right)}_{j_i^T}$$

$$\Box A_i^T = -\frac{4\pi}{c} j_i^T$$

Transvers. Strom ist quellenfrei // lektrisch auf

$\mathrm{div}\, j^T = \emptyset$ // mache $\partial_i j_i^T$ und

es folgt Kontinuitäts-Gl

raus

$\partial_i A_i = \emptyset \longrightarrow$ Fourier Raum : $\vec{k} \cdot \vec{A} = \emptyset$

$\rightarrow \vec{A} = \vec{A}^T$ (transversal im Ortsraum)

$\vec{B} = \mathrm{rot}\, \vec{A}^T \rightarrow \vec{B} = \vec{B}^T$ // ka warum?

$$\vec{E} = \vec{E}^T + \vec{E}^c$$

$$= -\frac{1}{c}\partial_t \vec{A}^T - \partial_i \phi_c$$

MW Gl aufschreiben f. Coulomb & transvers.

$$\partial_i B_i = \emptyset$$
$$\varepsilon_{ijk}\partial_j B_k = \frac{4\pi}{c} j_i + \frac{1}{c}\partial_t E_i$$
} B-Feld transv.

$$\varepsilon_{ijk}\partial_j E_k^T = -\frac{1}{c}\partial_t B_i$$ // fällt nicht weg
$$\varepsilon_{ijk}\partial_j E_k^c = \emptyset$$
$$\partial_i E_i^T = \emptyset$$ // wegen $\partial_i A_i^T$
$$\partial_i E_i^c = 4\pi\rho$$

$$A_i' = A_i - \partial_i \psi$$ // damit $\vec{B}$-Feld gleich

$$\phi' = \phi + \frac{1}{c}\partial_t \psi$$ ← // damit $\vec{E}$-Feld gl. bleibt $+ \frac{1}{c}\partial_t \ldots$

Eichbed.: $\partial_i A_i^T = \emptyset$

$$\partial_i A_i - \partial_i\partial_i \psi = \emptyset \qquad \Delta\psi = \operatorname{div}\vec{A} \quad ✓$$

umeichbar mit $\Delta\psi = \emptyset$

Fourier Trafo d. Felder

$$E(\vec{r},t) = \frac{1}{(2\pi)^4}\int d^3k\, d\omega\, E(\vec{k},\omega)\, e^{i\vec{k}\vec{r}}\, e^{-i\omega t}$$

Umkehrtrafo:

$$E(\vec{k},\omega) = \int d^3r\, dt\, E(\vec{r},t)\, e^{-i\vec{k}\vec{r}}\, e^{i\omega t}$$

Maxwellgl. Anfor:

$$\partial_i B_i(\vec{r},t) = \emptyset \longrightarrow \vec{k}\,\vec{B}(\vec{k},\omega) = \emptyset$$

$$\text{rot}\,\vec B = \frac{4\pi}{c}\vec j + \frac{1}{c}\partial_t \vec E \longrightarrow i\vec k \times \vec B(\vec k,\omega) = \frac{4\pi}{c}\vec j - \frac{1}{c} i\omega \vec E(\vec k,\omega)$$

$$/:i \quad \vec k \times \vec B = -\frac{4\pi i}{c}\vec j - \frac{\omega}{c}\vec E \quad \checkmark$$

$$\partial_i E_i = 4\pi\rho \longrightarrow i\vec k\,\vec E = 4\pi\rho \quad /:i \quad \checkmark$$

$$\text{rot}\,\vec E = -\frac{1}{c}\partial_t \vec B \qquad \cancel{i}\vec k \times \vec E = +\frac{\cancel{i}\omega}{c}\vec B \quad \checkmark$$

4 ω bzw. neg VZ in exp!

Kont. Gl:

$$\partial_i j_i + \partial_t \rho = \emptyset$$

$$\rightarrow \cancel{i}\vec k\,\vec j - \cancel{i}\omega\,\rho = \emptyset$$

Symmetrien: reelle Vektorf.

$$\vec E^*(\vec r,t) = \vec E(\vec r,t)$$

$$\vec E^* = \int d^3k\,d\omega\; E(\vec k,\omega)^* e^{-i\vec k\vec r} e^{i\omega t} = \int d^3k\,d\omega\; E(\vec k,\omega)\, e^{i\vec k\vec r} e^{-i\omega t}$$

hier $dk' = -dk$, $d\omega' = -d\omega$ } VZ 4 heben sich auf

dann wieder auf k umbenennen

$$= \int d^3k\,d\omega\; \vec E(-\vec k,-\omega)\, e^{-i\vec k\vec r} e^{i\omega t}$$

$$\rightarrow \quad E(\vec k,\omega)^* = E(-\vec k,-\omega)$$

$\rightarrow$ Übergang auf pos. Frequ.

$$\vec E(\vec r,t) = \frac{1}{(2\pi)^4}\int d^3k\,d\omega\,\vec E(\vec k,\omega)\, e^{i\vec k\vec r} e^{-i\omega t}$$

$$= c\int_{-\infty}^{0} d\omega \int d^3k + c\int_{0}^{\infty} d\omega \int d^3k$$

$$= \frac{1}{(2\pi)^4} \int_{-\infty}^{0} d\omega \int d^3k\, E(\vec{k},\omega)\, e^{i\vec{k}\vec{r}} e^{-i\omega t}$$

$$+ \frac{1}{(2\pi)^4} \int_{0}^{\infty} d\omega \int d^3k\, E(\vec{k},\omega)\, e^{i\vec{k}\vec{r}} e^{-i\omega t}$$

umwandeln: $d\omega' = -d\omega$

$dk' = -dk$

$$\frac{1}{(2\pi)^4} \int_{0}^{\infty} d\omega \int d^3k\, \vec{E}(\vec{k},-\omega)\, e^{-i\vec{k}\vec{r}} e^{i\omega t}$$

$$\rightarrow \vec{E}(\vec{r},t) = \frac{1}{(2\pi)^4} \int_{0}^{\infty} d\omega \int d^3k\, E(\vec{k},\omega)\, e^{i\vec{k}\vec{r}} e^{-i\omega t} + c.c.$$

Stimmt Trafo? Vorzeichen

Greensche Funktionen dafür

$$\Box D = -4\pi\, \delta(\vec{r}-\vec{r}\,')\cdot\delta(t-t')$$

löst:

$$\vec{A}(\vec{r},t) = \frac{1}{c} \int d^3r'\, dt'\, \vec{j}(\vec{r}\,',t')\, D(\vec{r},\vec{r}\,',t,t')$$

$$\phi(\vec{r},t) = \int d^3r'\, dt'\, \rho(\vec{r}\,',t')\, D(\vec{r},\vec{r}\,',t,t')$$

Berechnen: gehe in Fourier Raum mit D

$$\begin{cases} \vec{r} = \vec{r} - \vec{r}\,' \\ t = t - t' \end{cases}$$

$$D(\vec{r},t) = \frac{1}{(2\pi)^4} \int d^3k\, d\omega\, D(\vec{k},\omega)\, e^{i\vec{k}\vec{r}} e^{-i\omega t}$$

$$\delta(\vec{r},t) = \frac{1}{(2\pi)^4} \int d^3k\, d\omega\, e^{i\vec{k}\vec{r}} e^{-i\omega t}$$

$$\delta(\vec{k},\omega) = \int d^3\vec{r}\,' dt'\, \delta(\vec{r}-\vec{r}\,')\, \delta(t-t')$$

$$= 1$$

$$\Box D = -4\pi\, \delta(\vec{r}-\vec{r}\,')\, \delta(t-t')$$

Trafo nach Fourier:

$$\Box = \Delta - \frac{1}{c^2}\partial_t^2$$

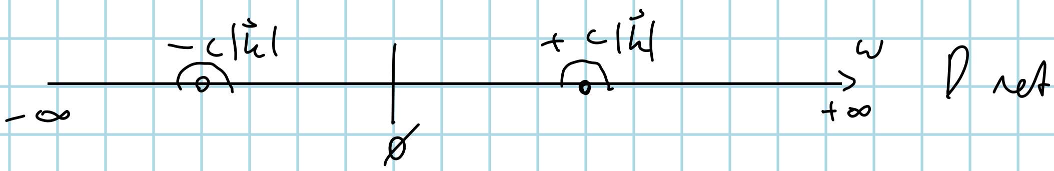
D soll nur von diff. abhängen!

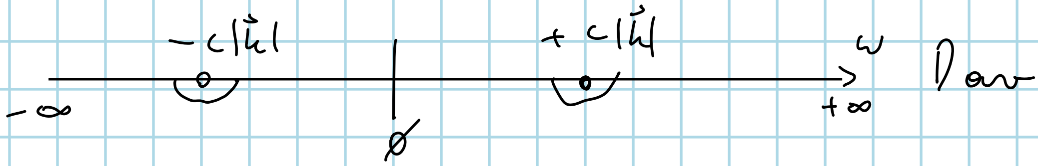
$$i^2\left(k^2 - \frac{\omega^2}{c^2}\right) D = -4\pi$$

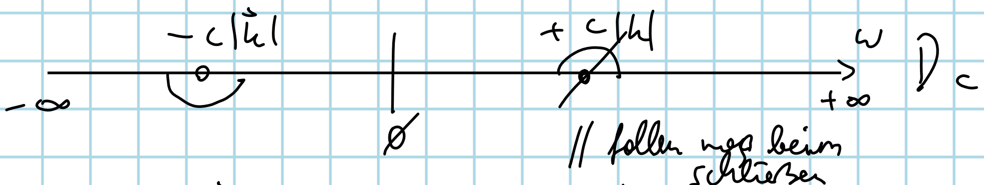
$$\rightarrow \quad D(\vec{k},\omega) = \frac{4\pi}{k^2 - \frac{\omega^2}{c^2}}$$

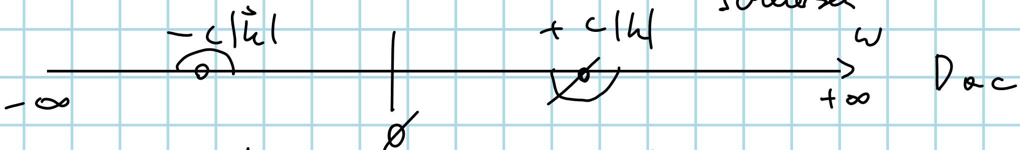
Rücktrafo: Pole bei $k^2 = \frac{\omega^2}{c^2}$

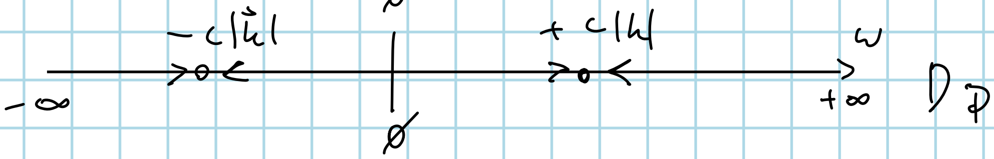
$$\omega = \pm c|\vec{k}|$$











Man will Pole umgehen → imaginär dazuzählen → $\omega' = \omega + i\eta$

ρ oben herum

$$D(\vec{k},\omega) = \frac{4\pi}{k^2 - \frac{\omega^2}{c^2}}$$

ret : 2x oben neg pol / pos pol

$$k^2 - \frac{\omega^2}{c^2} = \left(k + \frac{\omega}{c}\right)\left(k - \frac{\omega}{c}\right)$$

$$= \lim_{\eta \to 0} \left(k + \left(\frac{\omega}{c} + i\eta\right)\right)\left(k - \left(\frac{\omega}{c} + i\eta\right)\right)$$

$$= \text{—"—} \quad \left(k^2 - \left(\frac{\omega}{c} + i\eta\right)^2\right)$$

$$D_{ret} = \frac{4\pi}{\left(k^2 - \left(\frac{\omega}{c} + i\eta\right)^2\right)}$$

ave : 2x unten

$$D_{ave} = \frac{4\pi}{\left(k^2 - \left(\frac{\omega}{c} - i\eta\right)^2\right)}$$

c : 1x unten ~~1x oben~~ fällt weg

$$D_c = \frac{4\pi}{k^2 - \frac{\omega^2}{c^2} - i\eta}$$

ac : 1x oben ~~1x unten~~

$$D_{ac} = \frac{4\pi}{k^2 - \frac{\omega^2}{c^2} + i\eta}$$

P : $D_P = P \dfrac{4\pi}{k^2 - \frac{\omega^2}{c^2}} = \frac{1}{2}\left(D_{ret} + D_{ave}\right)$

Cauchy Hauptwert

Rücktrafo von D_{ret} :

$$D_{ret}(\vec{k}, \omega) = \frac{4\pi}{k^2 - \left(\frac{\omega}{c} + i\eta\right)^2}$$

$$D_{ret}(\vec r - \vec r\,', t - t') = \frac{1}{(2\pi)^4} \int d^3k \; e^{i\vec k(\vec r - \vec r\,')} \int d\omega \; e^{-i\omega(t-t')} \frac{4\pi}{k^2 - \frac{\omega^2}{c^2}}$$

Es bleibt gleich f $\int$, man muss nur wissen, wo das Res drin ist!

Vereinf: $\vec r - \vec r\,' := \vec r$ $\quad t - t' := t$

$$D_{ret}(\vec r, t) = \frac{2}{(2\pi)^3} \int d^3k \; e^{i\vec k \vec r} \overbrace{\int d\omega \; e^{-i\omega t} \frac{1}{k^2 - \frac{\omega^2}{c^2}}}^{F}$$

ω aufteilen $\quad \omega = \omega_r + i\omega_i \quad \longrightarrow \quad e^{-i\omega_r t} \; e^{\omega_i t}$

① ②

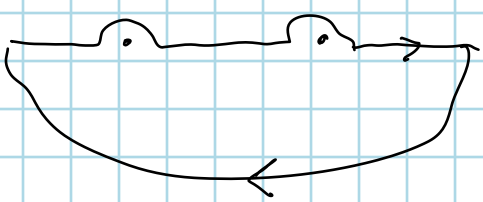
$\int$ muss endlich bleiben bzw Hilfspfad verschwinden

① schwingt $\rightarrow$ egal

② f $t > 0$: $\omega_i \rightarrow -\infty$ unten rum

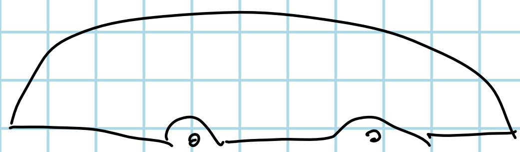
f $t < 0$: $\omega_i \rightarrow +\infty$ oben rum

// lässt Hilfsweg verschwinden



$t > 0$

beide drin



$t < 0$

keins drin

Nur f $t > 0$ Beitrag!

math negativ $\rightarrow$ neg UZ

später!

Res f. Pol 1. Ordnung

$\mathrm{Res}_a f(z) = (z-a)\, f(z)$

$$g = \frac{1}{f} = k^2 - \frac{\omega^2}{c^2} = \left(k + \frac{\omega}{c}\right)\left(k - \frac{\omega}{c}\right) = \frac{1}{c^2}(ck + \omega)(ck - \omega)$$

/ $e^{-i\omega t}$ bei f fehlt!

$$\operatorname*{Res}_{\omega=+ck} = \lim_{\omega\to ck} \cancel{(\omega - ck)}\, c^2 \frac{1}{-(ck+\omega)\cancel{(-ck+\omega)}} e^{-i\omega t}$$

kürzt sich dann — rausheben

$$= -c^{\cancel{2}} e^{-ickt} \frac{1}{2\cancel{c}k} = -\frac{c}{2k} e^{-ickt} \quad ✓$$

$$\operatorname*{Res}_{\omega=-ck} = \lim_{\omega\to -ck} \cancel{(\omega+ck)}\, c^2 \frac{1}{\cancel{(ck+\omega)}(ck-\omega)} e^{-i\omega t}$$

$$= e^{ickt} \frac{c}{2k} \quad ✓$$

$$\rightarrow \int d\omega\, e^{-i\omega t} \frac{1}{k^2 - \frac{\omega^2}{c^2}} = -\Theta(t) \frac{\cancel{2}\pi i c}{\cancel{2}k}\left(e^{ickt} - e^{-ickt}\right)$$

weil res nur f $t>0$ gelten

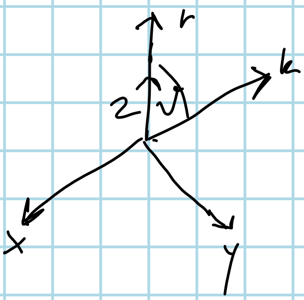
$$= -\frac{\pi i c}{k}\Theta(t)\left(e^{ickt} - e^{-ickt}\right) \quad ✓$$

jetzt $\int$ über $\vec{k}$

$$D(\vec{r},t) = -\frac{\cancel{2}}{(2\pi)^{\cancel{3}2}} \int d^3k\, \Theta(t) \frac{\cancel{\pi} i c}{k}\left(e^{ickt} - e^{-ickt}\right) e^{i\vec{k}\vec{r}}$$

$$= -\frac{ic}{(2\pi)^2} \int d^3k\, \Theta(t) \frac{1}{k}\left(e^{ickt} - e^{-ickt}\right) e^{i\vec{k}\vec{r}}$$

z Achse in Richtung $\vec{r}$ gelegt & Kugelkoord



$$\longrightarrow \vec{k}\vec{r} = kr\cos\vartheta$$

$$\rightarrow -\frac{ic}{(2\pi)^2}\int dk\, d\varphi\, d\vartheta\, k^2 \sin\vartheta\ \Theta(t)\frac{1}{k}\left(e^{ickt}-e^{-ickt}\right)$$

$$\cdot\, e^{ikr\cos\vartheta}$$

$k: 0-\infty$

$\varphi: 0-2\pi \qquad \vartheta: 0-\pi$

$$\xi = \cos\vartheta \qquad \frac{d\xi}{d\vartheta} = -\sin\vartheta$$

$$d\vartheta = -\frac{d\xi}{\sin\vartheta}$$

$$\rightarrow -\frac{ic}{(2\pi)^2}\int dk\, d\varphi\, d\xi\ k\ \Theta(t)\left(-e^{ickt}+e^{-ickt}\right)e^{ikr\xi}$$

$$\int_1^{-1} d\xi\ e^{ikr\xi} = \frac{e^{ikr\xi}}{ikr}\Bigg|_1^{-1} = \frac{1}{ikr}\left(e^{-ikr}-e^{ikr}\right)$$

$$\rightarrow -\frac{\cancel{i}c}{(2\pi)^2}\int dk\, d\varphi\ \cancel{k}\ \Theta(t)\left(e^{ickt}-e^{-ickt}\right)\left(e^{ikr}-e^{-ikr}\right)$$

$$\cdot\frac{1}{\cancel{ik}r}$$

$$= -\frac{c}{(2\pi)^{\cancel{2}}r}\int_0^\infty dk\ \cancel{2\pi}\ \Theta(t)\Big(e^{ik(r+ct)}+e^{-ik(r+ct)}$$

$$-e^{-ik(r-ct)}-e^{ik(r-ct)}\Big)$$

$$= -\frac{c\,\Theta(t)}{2\pi r}\int_{-\infty}^{\infty} dk\left(e^{ik(r+ct)}-e^{ik(r-ct)}\right)$$

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\ e^{ikx}$$

$$\rightarrow -\frac{c\,\Theta(t)}{r}\left(\delta(r+ct)-\delta(r-ct)\right)$$

$$\delta(ax) = \frac{\delta(x)}{a} \longrightarrow \delta\!\left(c\left(t+\frac{r}{c}\right)\right) = \frac{\delta\left(t+\frac{r}{c}\right)}{c}$$

gilt bei $t = -\frac{r}{c}$

$t > 0 \longrightarrow$ nur 2. Term $\qquad \delta(x) = \delta(-x)$

$\Longrightarrow$

$$\frac{c\,\Theta(t)}{r}\,\delta(r - ct) = D(\vec r, t)$$

$$= \frac{\cancel{c}\,\Theta(t)}{r}\,\frac{\delta(t - \frac{r}{c})}{\cancel{c}}$$

$$\Longrightarrow D(\vec r - \vec r', t - t') = \frac{\delta\left(t - t' - \frac{|r - r'|}{c}\right)}{|r - r'|}\,\Theta(t - t')$$

$$= \frac{\delta\left(t' - t + \frac{|r - r'|}{c}\right)}{|r - r'|}\,\Theta(t - t')$$

retardierte Potentiale:

$$\phi = \int d^3r'\,dt'\,G(\vec r - \vec r', t - t')\,\rho(\vec r', t')$$

$$\vec A = \frac{1}{c}\int d^3r'\,dt'\,G(\vec r - \vec r', t - t')\,\vec j(\vec r', t')$$

$$\phi = \int d^3r'\,dt'\,\frac{\delta\left(t' - t + \frac{|r - r'|}{c}\right)}{|r - r'|}\,\Theta(t - t')\,\rho(\vec r', t')$$

$$= \int d^3r'\,\frac{\rho\left(\vec r', t - \frac{|r - r'|}{c}\right)}{|r - r'|} \checkmark$$

// Θ ist hier eig. egal braucht man nur um 2. Term d. Greens zu killen

$\vec A$ analog

Die 2 Gl. gelten bei d. Lorenz Eichung & Pot. auch

Lsg gelten f. nat. RB. (normale D)

sonst $\quad D + D_{hom} \qquad \Rightarrow D_{hom} = \emptyset$

oder $\Box \phi_{hom} = \emptyset$ & $\Box \vec{A}_{hom} = \emptyset$

müssen $\partial_i A_i^h + \frac{1}{c}\partial_t \phi^h = 0$ erfüllen

Energie- & Impulsbilanz

Teilchen mit Ladung q

Energiesatz: $F = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$

$$dA = \vec{F}\,d\vec{s} = q\,\vec{E}\,d\vec{s} + q\,\frac{\vec{V}}{c}\times\vec{B}\,d\vec{s}$$

$$= q\,\vec{E}\,\vec{v}\,dt + \underbrace{\frac{q}{c}\underbrace{(\vec{v}\times\vec{B})}_{\perp\vec{v}}\vec{v}\,dt}_{\emptyset}$$

$$\frac{dA^{mech}}{dt} = q\,\vec{E}\,\vec{v} = \vec{j}\,\vec{E}$$

$\vec{j}\,\vec{E}$ aus MWG

$$\operatorname{rot}\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\partial_t\vec{E}$$

$$\left.\begin{aligned}\operatorname{rot}\vec{B} - \frac{1}{c}\partial_t\vec{E} &= \frac{4\pi}{c}\vec{j} \quad /\cdot\vec{E}\\ \operatorname{rot}\vec{E} + \frac{1}{c}\partial_t\vec{B} &= \emptyset \quad /\cdot\vec{B}\end{aligned}\right] -$$

$$\vec{E}\operatorname{rot}\vec{B} - \vec{E}\,\frac{1}{c}\partial_t\vec{E} - \vec{B}\operatorname{rot}\vec{E} - \vec{B}\,\frac{1}{c}\partial_t\vec{B} = \frac{4\pi}{c}\vec{j}\,\vec{E}$$

$$\vec{E}\operatorname{rot}\vec{B} - \vec{B}\operatorname{rot}\vec{E} - \vec{E}\,\frac{1}{c}\partial_t\vec{E} - \vec{B}\,\frac{1}{c}\partial_t\vec{B} = \frac{4\pi}{c}\vec{j}\,\vec{E}$$

$$\partial_i(\vec{E}\times\vec{B})_i - \frac{1}{2c}\partial_t\left(\vec{E}^2 + \vec{B}^2\right) = \frac{4\pi}{c}\vec{j}\,\vec{E} \quad /\cdot\frac{c}{4\pi}$$

$$\frac{c}{4\pi}\partial_i(\vec{E}\times\vec{B})_i - \frac{1}{8\pi}\partial_t\left(\vec{E}^2 + \vec{B}^2\right) = \vec{j}\,\vec{E}$$

$$\partial_i S_i - \partial_t w = \vec{j}\,\vec{E}$$

Für Vol. V statt Punkt

$$\frac{dA^{mech}}{dt} = \int_V d^3r \, \underbrace{\rho(\vec{r},t)\, v(\vec{r},t)}_{j(\vec{r},t)} \, E(\vec{r},t)$$

$$\rightarrow \frac{dA^{mech}}{dt} = \int d^3r \left(-\partial_i \underset{\text{Pointing V.}}{S_i} - \partial_t \underset{\text{Energiedichte}}{w}\right)$$

// Nr: $\vec{E} \operatorname{rot} \vec{B} - \vec{B} \operatorname{rot} \vec{E} = E_i \, \varepsilon_{ijk} \partial_j B_k - B_i \, \varepsilon_{ijk} \partial_j E_k$

$$= \varepsilon_{ijk} \left(E_i \partial_j B_k - B_i \partial_j E_k\right)$$

$$\varepsilon_{ijk} E_i \partial_j B_k = -\varepsilon_{kji} E_i \partial_j B_k$$
$$- \varepsilon_{ijk} E_k \partial_j B_i$$

$$\rightarrow -\varepsilon_{ijk} \left(E_k \partial_j B_i + B_i \partial_j E_k\right)$$
$$= -\varepsilon_{ijk} \left(\partial_j (E_k B_i)\right)$$
$$= -\varepsilon_{jki} \left(\partial_j (E_k B_i)\right) \quad \checkmark$$

$$\frac{dA^m}{dt} = -\int d^3r \, \partial_i S_i - \partial_t \underbrace{\int d^3r \, w}_{A^{feld}}$$

$$\frac{d}{dt} \underbrace{\left(A^{mech} + A^{feld}\right)}_{\text{ges } E \text{ im Vol.}} = -\int_V d^3r \, \partial_i S_i$$

$$= -\oint d^2\vec{f} \, S_i$$

Durch Oberfl. tretenden Energiestrom

Impulssatz d Edyn

$$\vec{F}^{mech} = \frac{d}{dt}\vec{P}^{mech} = \int_V d^3r \left(\rho(\vec{r},t)\,\vec{E}(\vec{r},t) + \rho\left(\frac{\vec{v}}{c}\times\vec{B}\right)\right)$$

MWGL:

$$\operatorname{div}\vec{B} = \emptyset \qquad / \cdot B$$

$$\operatorname{rot}\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\partial_t\vec{E} \qquad / \times\vec{B}$$

$$\operatorname{div}\vec{E} = 4\pi\rho \qquad / \cdot E$$

$$\operatorname{rot}\vec{E} = -\frac{1}{c}\partial_t\vec{B} \qquad / \times\vec{E}$$

$+$ $\quad +$

1) $\rho\vec{E}$:

$$E_i\partial_k E_k + B_i\partial_k B_k = 4\pi\rho\vec{E} \qquad / : 4\pi$$

1) $\vec{j}\times\vec{B}$:

$$\varepsilon_{ijk}\,\varepsilon_{jlm}(\partial_l B_m)B_k + \varepsilon_{ijk}\,\varepsilon_{jlm}(\partial_l E_m)E_k$$

$$= \frac{4\pi}{c}\varepsilon_{ijk}\,j_j\,B_k + \frac{1}{c}\left(\partial_t\,\varepsilon_{ijk}E_j\right)B_k$$

$$-\frac{1}{c}\left(\partial_t\,\varepsilon_{ijk}B_j\right)E_k$$

$$\varepsilon_{ijk}\,\varepsilon_{jlm} = \varepsilon_{jki}\,\varepsilon_{jlm} = (\delta_{kl}\delta_{im} - \delta_{km}\delta_{il})$$

$$\rightarrow (\partial_k B_i)B_k - (\partial_i B_k)B_k + (\partial_k E_i)E_k - (\partial_i E_k)E_k$$

$$\underbrace{-\frac{1}{c}(\dot{\vec{E}}\times\vec{B}) + \frac{1}{c}(\dot{\vec{B}}\times E)}_{-\frac{1}{c}\partial_t(\vec{E}\times\vec{B})} = \frac{4\pi}{c}\vec{j}\times\vec{B} \qquad / \cdot\frac{1}{4\pi}$$

$$\frac{1}{4\pi}\Big(E_i\partial_k E_k + E_k\partial_k E_i - E_k\partial_i E_k + \text{— " —}$$

$$-\frac{1}{c}\partial_t(\vec{E}\times\vec{B})\Big) = \rho E + \frac{\vec{j}}{c}\times\vec{B}$$

$$\frac{1}{4\pi}\left(\partial_k (E_i E_k) + \partial_k (B_i B_k) - \frac{1}{2}\partial_i (\vec{E}^2 + \vec{B}^2) - \frac{1}{c}\partial_t (\vec{E}\times\vec{B})\right)$$

$$= \rho \vec{E} + \frac{\vec{j}}{c}\times\vec{B}$$

$$\rightarrow \quad \partial_k \underbrace{\frac{1}{4\pi}\left(E_i E_k + B_i B_k - \frac{1}{2}\delta_{ik}(\vec{E}^2 + \vec{B}^2)\right)}_{\substack{T \\ \text{Spannungstensor}}} - \partial_t \underbrace{\frac{1}{4\pi c}(\vec{E}\times\vec{B})}_{\substack{\vec{g} \\ \text{Impulsdichte}}}$$

$$\rightarrow \quad \frac{d}{dt}\vec{p}^{\,m} = \int_V \left(\rho\vec{E} + \frac{\vec{j}}{c}\times\vec{B}\right) d^3r$$

$$= \int_V \partial_k T_{ik} - \partial_t g_i$$

$$= \int_V \partial_k T_{ik} - \partial_t \underbrace{\int_V g_i}_{\vec{p}^{\,f}}$$

$$\frac{d}{dt}\left(\vec{p}^{\,m} + \vec{p}^{\,f}\right) = \oint d^2 f \, \overleftrightarrow{T} \qquad \checkmark$$

III E-Statik

// $\partial_i B_i = 0$ $\qquad$ $\mathrm{rot}\,\vec{B} = \frac{4\pi}{c}\vec{j}$

$\rightarrow \partial_i E_i = 4\pi\rho$ $\qquad$ $\mathrm{rot}\,\vec{E} = 0$

$\mathrm{rot}\text{-}\mathrm{grad}\,\phi = 0 \quad \rightarrow \quad \vec{E} = -\partial_i \phi$

$\rightarrow \quad \Delta\phi = -4\pi\rho$

Lsg mit Greens

$$\phi = \int d^3r' \, G(\vec r - \vec r') \, \rho(\vec r')$$

$$G = \frac{1}{|\vec r - \vec r'|} \qquad \Delta G = -4\pi\delta$$

$\Delta \phi_{hom} = \emptyset$ od. $\Delta G_{hom} = \emptyset$

dann $G + G_{hom} = G_{spez.}$ statt G

RB auf geschl. Flächen

Entweder Dirichl. od. Neumann RB

funkt. $= \emptyset$ Ableitg $= \emptyset$ | am Rand

Aussage: Pot. im Inneren ist d. Pot am Rand eind. gegeben

Annahme: ϕ_1 & ϕ_2 wären 2 Lsg.

$$\chi = \phi_1 - \phi_2 \qquad \Delta\chi = \Delta\phi_1 - \Delta\phi_2$$

$$= 4\pi\rho - 4\pi\rho = \emptyset$$

Dirichlet: $\phi_1|_{\partial V} = \phi_2|_{\partial V} \rightarrow \chi = \emptyset$ am Rand

$$\int_V \vec\nabla(\chi\vec\nabla\chi) = \int_V \underbrace{\vec\nabla\chi \cdot \vec\nabla\chi}_{(\vec\nabla\chi)^2} + \chi \underbrace{\Delta\chi}_{\emptyset}$$

$$\int_{\partial V} \chi \vec\nabla\chi = \int_V (\vec\nabla\chi)^2 \rightarrow$$

$\emptyset$ am Rand

$$\int_V (\nabla\chi)^2 = \emptyset$$

$$\longrightarrow \vec\nabla\chi = \emptyset \text{ in } V$$

Entweder $\chi = \emptyset$ oder const

$\Delta \chi = \emptyset$ weil ja ist in V

$\partial_i \chi = 0$ am Rand & in V

aber χ kann const sein

Spezielle Green's G_D & G_N

$$\int_V \underbrace{G \Delta \phi}_{-4\pi\rho} - \underbrace{\phi \Delta G}_{-4\pi\delta} = \int_F G \vec{\nabla} \phi - \phi \vec{\nabla} G$$

$$-\int_V G 4\pi\rho + \underbrace{\int_V \phi \, 4\pi\delta}_{+4\pi\int d^3r' \, \phi(\vec{r}') \, \delta(\vec{r}-\vec{r}')} = \int_F G \vec{\nabla} \phi - \phi \vec{\nabla} G$$

$\rightarrow$ $/ : 4\pi$

$$\rightarrow \phi(\vec{r}) = \int_V d^3r' \, G(\vec{r}-\vec{r}') \, \rho(\vec{r}') + \int_F d^2f \, G(\vec{r}-\vec{r}') \, \vec{\nabla}\phi(\vec{r}') - \phi(\vec{r}') \, \vec{\nabla} G(\vec{r}-\vec{r}')$$

Dirichlet: $\phi|_F = \emptyset$

$$G_D(\vec{r}-\vec{r}')\big|_F = \emptyset$$

Neumann: $\frac{\partial \phi}{\partial n}\Big|_F = 0$

$\vec{\nabla} G|_F$ eig $= \emptyset$ geht aber nicht

weil sonst $\int_V \Delta G = \int -4\pi\delta$

$\int_F \nabla G = -4\pi$ nicht erfüllt!

$$\rightarrow \vec{\nabla} G|_F = -\frac{4\pi}{|F|}$$

Term ist const und egal f. $\vec{E}$

RB bei Leitern

Im Leiter: $\vec{E} = \emptyset$ f. zeitunabhängigen Zustand im Leiter

$\phi = \text{const}$

Ladungen sind auf Oberfl. — $\partial_i E_i = 4\pi\rho$

$E_i = 4\pi\sigma$ auf Oberfl.

$$\int_V \partial_i E_i = \int 4\pi\rho$$
$$= 4\pi Q$$
$$\int_F E_i = 4\pi Q$$
$$F E_i = 4\pi Q$$
$$E_i = 4\pi \underbrace{\frac{Q}{F}}_{\sigma}$$

$\phi = \emptyset$ innen

RB: $\vec{E}_t = 0$ Rand

$E_n = 4\pi\sigma$

Entweder Q od ϕ vorgeben

Bildladungen

$\Delta \phi_{hom} = \emptyset$ zufügen f. RB

$$\phi = \int d^3r' \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} + \phi_{hom}$$

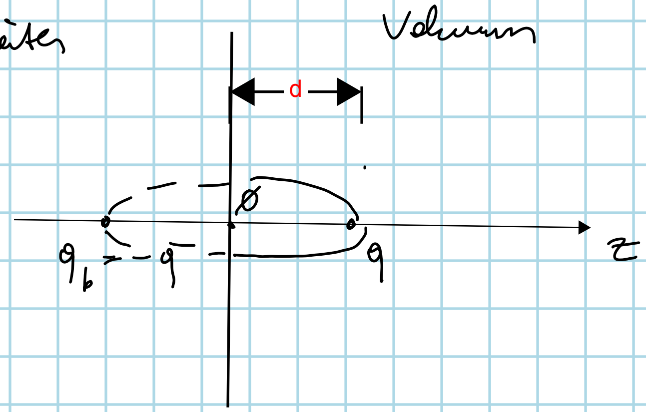
kann $\phi_{hom} = \phi_{bild} = \int d^3r' \dfrac{\rho^{bild}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$

wobei ρ^{bild} im Vol $\emptyset$ ist

$\rightarrow \Delta \phi = \emptyset$ im Vol ✓

ρ^{bild} so wählen, dass $\phi_b + \phi$ zu den RB passt!

zB Halbraum:



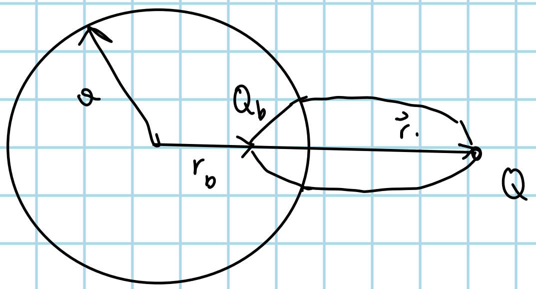
$$\rho_b(x,y,z) = -\rho(x,y,-z) \qquad // \text{ f. } z < \emptyset$$

Bsp: $\phi(\vec{r}) = \frac{q}{|\vec{r} - d\vec{e}_z|} - \frac{q}{|\vec{r} + d\vec{e}_z|} \qquad // |\vec{r} - \vec{r}'|$

f. E: ableiten!

Kugelförmig

$$Q_b = -Q \frac{a}{r}$$



$$r_b = a \frac{a}{r}$$

Leiter ungeladen, dann in die Mitte noch $-Q_b$ setzen

Pot. auf Oberfl.: $\phi = \frac{Q_{\text{mittelpt}}}{a} = -\frac{Q_b}{a}$

Leiter auf Q_0 Mitte: $Q_0 - Q_b$

$$\rightarrow \phi = \frac{Q_0 - Q_b}{a}$$

Leiter auf $\phi_0 \rightarrow$ $\mathcal{G}$ σ Q berechnen

$$\int_\Omega \sigma \, d\Omega = \phi_0 \, a + Q_{bild}$$

Elektr. Multipolentw.

Feld weit weg von lok. Quelle

$\rho(\vec{r}) = \emptyset$ f $|\vec{r}| > R$ (ausgedeh. Kugel)

$$\phi(\vec{r}) = \int d^3r' \frac{\rho(r')}{|\vec{r} - \vec{r}'|}$$

Entwickeln v $\frac{1}{|\vec{r} - \vec{r}'|}$ f kleine $\vec{r}'$

$$\frac{1}{\sqrt{r^2 - 2\vec{r}\vec{r}' + r'^2}} = \frac{1}{r\sqrt{1 \underbrace{- 2\frac{\vec{r}\vec{r}'}{r^2} + \frac{r'^2}{r^2}}_{x}}}$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$$

$$\rightarrow \frac{1}{r}\left(1 - \frac{1}{2}\left(-2\frac{\vec{r}\vec{r}'}{r^2} + \frac{r'^2}{r^2}\right) + \frac{3}{8}\left(\frac{4(\vec{r}\vec{r}')^2}{r^4} - 4\frac{\vec{r}\vec{r}'^3}{r^4} + \frac{r'^4}{r^4}\right)\right)$$

$$\rightarrow \frac{1}{r} + \frac{\vec{r}\vec{r}'}{r^3} - \frac{r'^2}{2r^3} + \frac{3}{\cancel{8}\,2}\frac{\cancel{4}(\vec{r}\vec{r}')^2}{r^5} - \frac{\cancel{12}\,3}{\cancel{8}\,2}\frac{\vec{r}\vec{r}'^3}{r^5} + \frac{3}{8}\frac{r'^4}{r^5}$$

$$\phi = \int \frac{\rho}{\underbrace{|\vec{r} - \vec{r}'|}}$$

$x_i x_i'$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{r} + \frac{\vec{r}\vec{r}'}{r^3} + \frac{1}{2}\frac{3(\vec{r}\vec{r}')^2 - r^2 r'^2}{r^5}$$ // r^2 rausheben

$$\rightarrow \phi(\vec r) = \frac{1}{r}\int d^3r'\,\rho(\vec r') + \frac{\vec r}{r^3}\int d^3r'\,\rho(\vec r')\,\vec r' + \frac{1}{2}\frac{x_i x_j}{r^5}\underbrace{\int d^3r'\,(3x_i'x_j' - \delta_{ij}r'^2)\rho(\vec r')}$$

$$\phi = \frac{q}{r} + \frac{\vec p\,\vec r}{r^3} + \frac{1}{2}\frac{x_i x_j}{r^5}Q_{ij}$$

Monopol (Coulomb) Dipol Quadrupol

$\vec E$ ausrechnen:

$$\vec E_1 = \frac{q\vec r}{r^3}$$

$$-\partial_i \frac{1}{r} = -\partial_i (x_i x_i)^{-\frac{1}{2}}$$

$$\rightarrow +\frac{1}{\cancel{2}}\cdot(\ldots)^{-\frac{3}{2}}\cdot \cancel{2}x_i$$

$$\rightarrow \frac{\vec r}{r^3}$$

$$\vec E_2 = -\partial_i \frac{\vec p\,\vec r}{r^3} = -\partial_i \frac{p_j r_j}{r^3} = -p_j\left(\frac{1}{r^3}\partial_i r_j + r_j\partial_i\frac{1}{r^3}\right)$$

$$= -p_j\left(\frac{\delta_{ij}}{r^3} - 3r_j\frac{r_i}{r^5}\right) = \frac{3r_i r_j p_j - r^2\delta_{ij}p_j}{r^5} \checkmark$$

alles f. weit entfernte Felder $r > R$

<u>Sphärische Multipole.</u>

Entw. in Kugelflächenfkt.

$$\frac{1}{|\vec r - \vec r'|} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y^*_{lm}(\vartheta_<,\varphi_<)\,Y_{lm}(\vartheta_>,\varphi_>)$$

$$r > r' \longrightarrow r^> = r \qquad r^< = r'$$

$$\rightarrow \phi(\vec r) = \sum_l\sum_m \frac{4\pi}{2l+1}\frac{Y_{lm}(\vartheta,\varphi)}{r^{l+1}}\underbrace{\int Y^*_{lm}(\vartheta',\varphi')\,r'^l\,d^3r'}_{q^*_{lm}\ \text{sphärische Multipole.}}$$

außerhalb

Symmetrie: $q^*_{l-m} = (-1)^m q_{lm}$

l. Moment Komponenten

Betrag d. l. Mom.:

$$|q_l| = \left(\sum_{m=-l}^{l} \frac{4\pi}{2l+1} |q_{lm}|^2 \right)^{\frac{1}{2}}$$

Mittelwert d. $\vec{E}$ im Kugelbereich

$$\langle \vec{E} \rangle = \frac{1}{V} \int_{\text{Kugel}} \vec{E}(\vec{r})\, d^3r = \frac{3}{4\pi R^3} \int \vec{E}(\vec{r})\, d^3r$$

$$= -\frac{3}{4\pi R^3} \int_{r<R} \partial_i \phi(\vec{r})\, d^3r = -\frac{3}{4\pi R^3} \int_{r<R} d^3r\, \partial_i \int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r'$$

$$= -\frac{3}{4\pi R^3} \oint_{r=R} d^2f \int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r' = -\int d^3r' \rho(\vec{r}') \underbrace{\frac{3}{4\pi R^3} \oint_{r=R} d^2f \frac{1}{|\vec{r}-\vec{r}'|}}_{\vec{I}(\vec{r}')}$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{r_<^l}{r_>^{l+1}} Y^*_{lm}(\vartheta_<, \varphi_<)\, Y_{lm}(\vartheta_>, \varphi_>) \frac{4\pi}{2l+1}$$

$$\oint d^2f = \int d\Omega\, \vec{e}_r \qquad e_r = \begin{pmatrix} \cos\varphi \sin\vartheta \\ \sin\varphi \sin\vartheta \\ \cos\vartheta \end{pmatrix}$$

$\int$ mit Kugelh., e_r' in z-Richtung

$$\vec{I}(\vec{r}') = I \cdot \vec{e}_{r'} \qquad \vec{e}_r\, \vec{e}_{r'} = \vec{e}_r \vec{e}_z = \cos\vartheta$$

ges!

$$I = \frac{3}{4\pi R^3} \sum_{lm} \int d\Omega\, R^2\, Y^*_{lm}(\vartheta_<, \varphi_<)\, Y_{lm}(\vartheta_>, \varphi_>) \frac{r_<^l}{r_>^{l+1}} \cos\vartheta \frac{4\pi}{2l+1}$$

$$= \frac{3}{4\pi R^3} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} R^2 \int d\Omega \text{ — " — } \frac{r_<^\ell}{r_>^{\ell+1}} \sqrt{\frac{4\pi}{3}}\, Y_{10} \frac{4\pi}{2\ell+1}$$

$$\int d\Omega\; Y^*_{\ell' m'}(\vartheta,\varphi)\, Y_{\ell m}(\vartheta,\varphi) = \delta_{\ell\ell'}\delta_{mm'}$$

$\oint_{r=R}$! $r > r'$:

$$I = C R^2 \sum_{\ell m} \int d\Omega\; Y^*_{\ell m}(\vartheta',\varphi')\, Y_{\ell m}(\vartheta,\varphi) \frac{r'^\ell}{r^{\ell+1}}\, Y^*_{10}(\vartheta,\varphi) \sqrt{\frac{4\pi}{3}} \cdot \frac{4\pi}{2\ell+1}$$

$$= C R^2 \sum_{\ell m} Y^*_{\ell m}(\vartheta',\varphi')\, \delta_{\ell 1}\delta_{m0} \frac{r'^\ell}{r^{\ell+1}} \frac{4\pi}{2\ell+1} \sqrt{\frac{4\pi}{3}}$$

$$= C R^2 Y^*_{10}(\vartheta',\varphi') \frac{r'}{r^2} \frac{4\pi}{3} \sqrt{\frac{4\pi}{3}}$$

$\cos\vartheta' = \cos\cancel{\varnothing}$

$= \cancel{\varnothing}$ weil z-Achse $r' = z$!!

$$= \frac{3\cancel{R^2}}{\cancel{4\pi} R^3} \frac{r'}{\cancel{R^2}} \frac{\cancel{4\pi}}{\cancel{3}} = \frac{r'}{R^3}$$

$$\boxed{\vec{I}\cdot\vec{e}_{r'} = I \;/\quad \vec{I} = I\,\vec{e}_{r'}}$$

$r < r'$:

$$I = C R^2 \sum \int d\Omega\; Y^*_{\ell m}(\vartheta,\varphi)\, Y_{\ell m}(\vartheta',\varphi') \frac{r^\ell}{r'^{\ell+1}}\, Y_{10}(\vartheta,\varphi) \sqrt{\frac{4\pi}{3}} \cdot \frac{4\pi}{2\ell+1}$$

$$= \frac{3}{4\pi R} \sum_{\ell m} \delta_{\ell 1}\delta_{m0} \frac{4\pi}{2\ell+1} \sqrt{\frac{4\pi}{3}}\, Y_{\ell m}(\vartheta',\varphi') \frac{r^\ell}{r'^{\ell+1}}$$

$$= \frac{\cancel{3}}{\cancel{4\pi} R} \frac{\cancel{4\pi}}{\cancel{3}} \underbrace{\sqrt{\frac{4\pi}{3}}\, Y_{10}(\vartheta',\varphi')}_{\cos(\cancel{\varnothing})} \frac{r}{r'^2} = \frac{\cancel{R}}{\cancel{R}\; r'^2}$$

$$\Longrightarrow \quad r < r': \frac{\vec{r}\,'}{R^3} \qquad r > r': \frac{\vec{r}}{r'^3}$$

$$\rightarrow \langle \vec{E} \rangle = -\underbrace{\int_{R<r'}}_{\text{außen}} d^3r'\, \rho(\vec{r}\,') \frac{\vec{r}\,'}{r'^3} - \underbrace{\int_{R>r'}}_{\text{innen}} d^3r'\, \rho(\vec{r}\,') \frac{\vec{r}\,'}{R^3}$$

1. Term: Ladungen außen erzeugen $\vec{E}$ in MP

2. Term: $\propto$ Dipolmoment innen

E – statische Energie

Energiedichte: $w = (E^2 + B^2) \cdot \frac{1}{8\pi}$

$$W = \frac{1}{8\pi} \int_V d^3r \, \vec{E}^2 = -\frac{1}{8\pi} \int_V d^3r \left(E_i \partial_i \phi \right)$$

$$\partial_i E = 4\pi \rho$$

$$\partial_i (E_i \phi) - (\phi \, \partial_i E_i)$$

$$\rightarrow \frac{1}{8\pi} \int_V \phi \, 4\pi \rho + \frac{1}{8\pi} \oint \phi \, \partial_i \phi$$

$$= \frac{1}{2} \int_V \phi \, \rho + \frac{1}{8\pi} \oint \phi \, \partial_i \phi$$

fällt weg bei nat. RB

$$\rightarrow W = \frac{1}{2} \int d^3r \int d^3r' \frac{\rho(\vec{r}') \, \rho(\vec{r})}{|\vec{r} - \vec{r}'|}$$

Aufteilung d. Quellen

$$\rho = \sum_n \rho_n \quad \& \quad \text{Pot.} \quad \phi = \sum_n \phi_n$$

$$\rightarrow W = \frac{1}{2} \int_V \phi(\vec{r}) \, \rho(\vec{r}) \qquad \Delta \phi_n = -4\pi \rho_n$$

$$= \frac{1}{2} \int_V \Big(\sum_n \phi_n \rho_n + \underbrace{\sum_{n<m} \phi_n \rho_m + \sum_{n>m} \phi_n \rho_m}_{2 \sum_{n<m} \phi_n \rho_m} \Big)$$

Symmetrie weil $\phi_n = \frac{q_m}{r}$

$\phi_m = \frac{q_n}{r}$

$$\Rightarrow \underbrace{\frac{1}{2}\int_V \sum_n^{n=m} \phi_n \rho_n}_{\text{Selbstenergie}} + \underbrace{\int_V \sum_{n<m} \phi_n \rho_m}_{\text{WW-Energie}}$$

$$W_{selbst} = \frac{1}{2} \sum_n \iint d^3r\, d^3r' \frac{\rho_n(\vec{r}')\, \rho_n(\vec{r})}{|\vec{r}-\vec{r}'|}$$

$$U_{ww} = \sum_{n<m} \iint d^3r\, d^3r' \frac{\rho_n(\vec{r}')\, \rho_m(\vec{r})}{|\vec{r}-\vec{r}'|}$$

Kapazitätskoeff

$$\phi(\vec{r}) = \int_V G \rho + \frac{1}{4\pi}\left(\oint_F G \nabla'\phi - \phi \nabla' G\right)$$

Pot. außen

Ladung nur am Rand

dazwischen $= \emptyset$

Dirichlet RB

$\phi|_F$ = vorgeg $\longrightarrow$ 2.T $= \emptyset$

weil $d^2 f$ in andere Richtg

$$\rightarrow \phi(\vec{r}) = + \int_F \phi \nabla' G$$

$\parallel$ $\vec{E} = 4\pi \sigma$

Gesamtladung $Q_m = \frac{1}{4\pi} \oint \vec{E}(\vec{r})\, d^2\vec{f} = -\frac{1}{4\pi} \oint \partial_i \phi(\vec{r}) d^2 f_i$

$$= \sum_n \underbrace{-\frac{1}{(4\pi)^2} \oint_{F_m} d^2 f_i\, \partial_i \oint_{F_n} d^2 f'_j \left(\partial_j G(\vec{r}, \vec{r}')\right)}_{C_{mn}} \phi_n(\vec{r}')$$

$$Q_m = \sum_n C_{mn} \phi_n \quad \checkmark$$

$$W = \frac{1}{2}\int_V \phi \rho - \underbrace{\frac{1}{8\pi} \oint \phi\, \partial_i \phi\, d^2\vec{f}}_{\text{weil nach innen}}$$

f. diesen Fall $= \emptyset$

$$W = -\frac{1}{8\pi} \oint \phi \underbrace{\partial_i \phi}_{-\vec{E}} = +\frac{1}{8\pi} \oint_{F_m} \sum_m \phi_m \vec{E}$$

$$= \frac{1}{8\pi} 4\pi \sum_m Q_m \phi_m$$

$$W = \frac{1}{2} \sum_m Q_m \phi_m \quad \checkmark$$

<u>Kondensator</u>

$$Q_1 = -Q_2 = Q$$

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{\det C_{mn}} \begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \quad // \text{C Matrix invertieren!}$$

$$C = \frac{Q}{\phi_1 - \phi_2} \qquad \phi_1 = \frac{1}{C_{11}C_{22} - C_{12}C_{21}} (C_{22} Q_1 - C_{12} Q_2)$$

$$\phi_2 = \frac{1}{-\text{"}-} (-C_{21} Q_1 + C_{11} Q_2)$$

$$\phi_1 - \phi_2 = \frac{1}{C_{11}C_{22} - C_{12}C_{21}} (C_{22} Q + C_{12} Q + C_{21} Q + C_{11} Q)$$

$$C = \frac{Q}{\phi_1 - \phi_2} = \frac{C_{22} + C_{12} + C_{21} + C_{11}}{C_{11} C_{22} - C_{12} C_{21}}$$

$$W = \frac{1}{2} \sum_m Q_m \phi_m = \frac{1}{2} (\phi_1 Q_1 + \phi_2 Q_2) =$$

$$\frac{Q}{2} (\phi_1 - \phi_2) \quad \checkmark$$

Lokalisierte LV im Feld

WW-Problem LV am Pkt $\vec{R}$

$\rho(\vec{r}-\vec{R})$ Feld $\vec{E}(\vec{r})$

$$W_{ww} = \int_V d^3r \; \phi(\vec{r}) \, \rho(\vec{r}-\vec{R}) \qquad \vec{r}\,' = \vec{r}-\vec{R}$$

$$\int_V d^3r \; \phi(\vec{R}+\vec{r}) \, \rho(\vec{r})$$

$$\phi(x) = \phi(x_0) + (\partial_i \phi(x_0))(x-x_0) + \frac{1}{2}(\partial_i \partial_j \phi(x_0))(x-x_0)^2$$

$$\phi(\vec{R}+\vec{r}) = \phi(\vec{R}) + (\partial_i \phi(\vec{R})) r_i + \frac{1}{2}(\partial_i \partial_j \phi(\vec{R})) r_i r_j$$

$$\rightarrow W_{ww} = \phi(\vec{R}) \underbrace{\int_V \rho(\vec{r})}_{q} + \partial_i \phi(\vec{R}) \underbrace{\int_V \rho \, r_i}_{p_i} + \frac{1}{2} \underbrace{\partial_i \partial_j \phi \int_V r_i r_j \rho}_{\text{auf } Q_{ij} \text{ ändern}}$$

$$Q_{ij} = \int (3x_i' x_j' - \delta_{ij} r'^2) \, \rho(\vec{r}\,') \, d^3r'$$

$$W_{ww} = \phi(\vec{R}) \, q - \vec{E}(\vec{R}) \, \vec{p} - \frac{1}{6} \frac{\partial}{\partial R_i} E_j \, Q_{ij}$$

$$\frac{\partial}{\partial R_i} E_j \, \delta_{ij} = \frac{\partial}{\partial R_i} E_i = \emptyset \qquad \operatorname{div} \vec{E} = \emptyset$$

kein ρ im Raum

WW zw. Dipole

Dipol p_1 im Ursprung

Poti in $\vec{R}$: $\phi(\vec{R}) = \frac{\vec{p}_1 \vec{R}}{R^3}$

$\vec{E}$ ausrechnen $\vec{E}(\vec{R}) = \frac{3(\vec{p}\vec{R})\vec{R} - R^2 \vec{p}}{R^5}$

$$W_{ww} = -\vec{p}_2 \vec{E}(\vec{R}) \nearrow$$ // 2. Dipol mit p_2

$$U = \frac{|\vec{p}_1||\vec{p}_2|}{R^3}$$ in W_{ww} einsetzen

f	$\vec{p}_1$	$\vec{p}_2$	$\longrightarrow$ $\vec{R}$
	$\rightarrow$	$\leftarrow$	1)
	$\uparrow$	$\uparrow$	
	$\rightarrow$	$\uparrow$	
	$\downarrow$	$\uparrow$	
	$\rightarrow$	$\rightarrow$	

1)
$$\frac{-R^2 p_1 p_2 + 3 R_{p_1} R_{p_2}}{R^5}$$

$$= \frac{2 p_1 p_2}{R^3} = 2W \checkmark$$

Kräfte in $\vec{E}$ - Feldern

/ auf lok. LV

$$\vec{F}(\vec{r}) = \int_V \rho(\vec{r} - \vec{R}) \, \vec{E}(\vec{r}) \longrightarrow$$

$$\int E(\vec{R} + \vec{r}) \, \rho(\vec{r}) \, d^3\vec{r}$$

$$E_i(R_j + r_j) = E_i(R_j) + \partial_k E_i(R_j) \, r_k$$

$$\vec{F} = \vec{E} \underbrace{\int_V \rho(\vec{r})}_{\vec{q}} + \underbrace{\frac{\partial}{\partial R_k} \vec{E}(\vec{R}) \underbrace{\int_V \rho(\vec{r}) \, r_k}_{p_k}}_{p_k \frac{\partial \vec{E}}{\partial R_k} \; - \; p_k \partial_k E_i}$$

$$\varepsilon_{ijk} \, p_j \underbrace{\varepsilon_{klm} \partial_l E_m}_{\varnothing \text{ statisch}} = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) \, p_j \partial_l E_m$$

$$p_m \partial_i E_m - p_l \partial_l E_i$$

kann man vorziehen da $p = \text{const}$

$$\rightarrow \vec{F}(\vec{R}) = q \vec{E}(\vec{R}) + \vec{\nabla}_R (\vec{p} \cdot \vec{E}(\vec{R}))$$

$$W_{ww} = -q\,\phi(\vec{R}) - p\,\vec{E}(\vec{R})$$

$$\rightarrow \quad \vec{F} = -\vec{\nabla} W_{ww}$$

Drehmoment:

$$\vec{N}(\vec{R}) = \vec{r} \times \vec{F} = \int_V \vec{r} \times \rho(\vec{r})\,\vec{E}(\vec{R}+\vec{r})$$

$$\vec{E}(\vec{R}+\vec{r}) = \vec{E}(\vec{R}) + \partial_i E_j\, r_i \; - - -$$

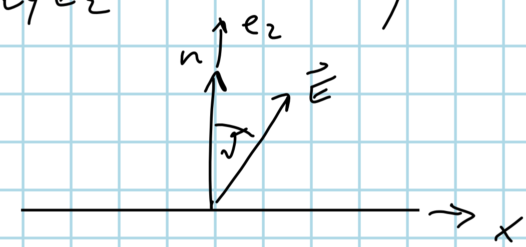
$$\rightarrow \int_V \vec{r} \times \rho\, \vec{E}(\vec{R}) + \ldots$$

$$\vec{N}(\vec{R}) = \vec{p} \times \vec{E}(\vec{R}) + \ldots$$

Maxwell Spannungstensor

$$T_{ij} = \frac{1}{4\pi}\left(E_i E_j + B_i B_j\right) - \frac{1}{8\pi}\left(E^2 + B^2\right)\delta_{ij}$$

$$\overleftrightarrow{T} = \frac{1}{4\pi}\begin{pmatrix} E_x E_x - E^2\frac{1}{2} & E_y E_x & \vdots \\ E_x E_y & E_y E_y - \frac{1}{2}E^2 & \vdots \\ E_x E_z & E_y E_z & \vdots \end{pmatrix}$$



Flächenkraft

$$\vec{T} = T_{ij}\, n_j = \frac{1}{4\pi}\begin{pmatrix} E_z E_x \\ E_z E_y \\ E_z E_z - E^2/2 \end{pmatrix}$$

$$n_j = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

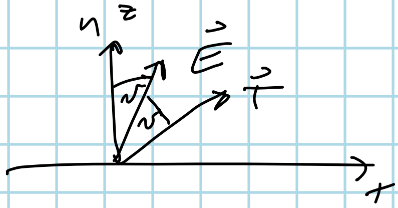
$$\vec{E} = E\begin{pmatrix} \sin\vartheta \\ \emptyset \\ \cos\vartheta \end{pmatrix} \qquad \vec{E} \text{ in } x-z \text{ Ebene}$$

$$\vec{T} = \frac{E^2}{4\pi}\begin{pmatrix} \cos\vartheta \sin\vartheta \\ \emptyset \\ \cos^2\vartheta - \frac{1}{2} \end{pmatrix}$$

$$2\sin\vartheta\cos\vartheta = \sin 2\vartheta$$

$$2\cos^2\vartheta - 1 = \cos 2\vartheta$$

$$\rightarrow \vec T = \frac{E^2}{8\pi}\begin{pmatrix}\sin 2\vartheta\\ 0\\ \cos 2\vartheta\end{pmatrix}$$



✓

Bewegung eines gel. Teilchens

$$F = qE \Rightarrow m\ddot{\vec x} = q\vec E \quad / \ \vec E_0$$

$$\ddot{\vec x} = \frac{q}{m}\vec E_0$$

$$\dot x = \frac{q}{m}E_0 t + v_0$$

$$x = \frac{q}{m}E_0\frac{t^2}{2} + v_0 t + x_0$$ ✓

4) Magnetostatik

Magn Felder sind rot. RB:

$$\partial_i B_i = 0 \rightarrow \vec B = \text{rot}\,\vec A$$

$$\text{rot}\,\vec B = \frac{4\pi}{c}\vec j + \cancel{\frac{1}{c}\partial_t\vec E}$$ // stat.

$$\varepsilon_{ijk}\partial_j\varepsilon_{klm}\partial_l A_m = (\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl})\partial_j\partial_l A_m$$

$$= \partial_m\partial_i A_m - \partial_l\partial_l A_i$$ ✓

$$\partial_i\partial_j A_j = \partial_j\partial_i A_j \ ?$$ // müsste passen

$$\rightarrow \text{grad}\,\underbrace{\text{div}\,\vec A}_{=0} - \Delta\vec A = \frac{4\pi}{c}\vec j$$

Coulomb Eichung

$$\Delta\vec A = -\frac{4\pi}{c}\vec j$$

$$\rightarrow \vec A = \frac{1}{c}\int_V d^3r'\,\frac{\vec j(\vec r')}{|\vec r-\vec r'|}$$

Für andere RB $\Delta A_n = 0$ (Anoddeien)

$\vec{B}$ berechnen!

$$\varepsilon_{ijk}\,\partial_j A_k = \frac{1}{c}\varepsilon_{ijk}\partial_j \int_V d^3r' \frac{j_k}{\left((x_i - x_i')^2\right)^{\frac{1}{2}}}$$

$$\rightarrow \vec{B} = \frac{1}{c}\int_V \frac{\vec{j} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d^3r'$$

Magn. Dipolmoment

$$j(\vec{r}) = \emptyset \qquad\qquad |\vec{r}| > R$$

$$\vec{A} = \frac{1}{c}\int_V d^3r' \frac{j(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$$ — Entwickeln

$$\frac{1}{|\vec{r} - \vec{r}\,'|} = \frac{1}{r} + \frac{\vec{r}\,\vec{r}\,'}{r^3} + \frac{1}{2}\,\frac{3(\vec{r}\,\vec{r}\,')^2 - r^2 r'^2}{r^5}$$

$$\rightarrow \vec{A} = \frac{1}{cr}\int_V d^3r'\, j(\vec{r}\,') + \frac{1}{cr^3}\int_V (r\vec{r}\,')\, j(\vec{r}\,')\, d^3r' + \dots$$

Kontinuitätsgl: $\partial_i j_i = \emptyset$

$$\partial_i'(j_i r_j') = r_j' \underbrace{\partial_i' j_i}_{\emptyset} + \underbrace{j_i \underbrace{\partial_i' r_j'}_{\delta_{ij}}}_{j_j}$$

$$\int_V d^3r'\, j(\vec{r}\,') = \int d^3r'\, \partial_i'(j_i r_j')$$

$$= \oint j_i r_j' \qquad // \text{ wat RB} = \emptyset$$

$\rightarrow$ 1. Term $= \emptyset$

$$\underbrace{\partial'_k(j_k r_i r'_i r'_l)}_{\text{Oberfl. fällt weg}} = \underbrace{\partial'_k(j_k r'_l)}_{j_l} r_i r'_i + \partial'_k(r_i r'_i) j_k r'_l$$

$$= j_l r_i r'_i + r_k j_k r'_l$$

soll sich aufheben

$$\varepsilon_{ijk} r_j \varepsilon_{klm} r'_l j_m = r_i j_j r'_i - r_j r'_j j_i$$

$$a \times (b \times c) = (a \cdot c) b - (a b) c$$

$$- c \times (a \times b) = -(c \cdot b) a + (c a) b \quad \checkmark$$

$$\varepsilon_{ijk} \partial_j \varepsilon_{klm} \partial_l A_m = \partial_j A_j \partial_i - \partial_j \partial_j A_i$$

$$\underbrace{\partial'_k(\ldots)}_{\emptyset} - r \times r' \times j = (r \cdot r') j + (r \cdot r') j + \cancel{(r \cdot j) r'} - \cancel{(r \cdot j) r'}$$

$$-\frac{1}{2} \vec{r} \times \vec{r}' \times \vec{j} = (\vec{r} \cdot \vec{r}') \vec{j}$$

$$\vec{A} = -\frac{1}{2 c r^3} \int \vec{r} \times \vec{r}' \times \vec{j} \, d^3 r'$$

$$\vec{m} = \int_V \frac{1}{2c} \vec{r}' \times \vec{j} \, d^3 r' \longrightarrow \vec{A} = \frac{1}{r^3} \quad \vec{m} \times \vec{r}$$

$$\vec{B} = \varepsilon_{ijk} \partial_j \frac{1}{r^3} \varepsilon_{klm} m_l r_m$$

$$= \varepsilon_{ijk} \varepsilon_{klm} \partial_j \frac{1}{r^3} m_l r_m$$

$$= (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \partial_j \frac{1}{r^3} m_l r_m$$

$$= \partial_m \frac{1}{r^3} m_i r_m - \partial_l \frac{1}{r^3} m_l r_i$$

$$= m_i \left(-\frac{3 r_m}{r^5} r_m + \frac{3}{r^3}\right) - m_l \left(-\frac{3 r_l}{r^5} r_i + \frac{\delta_{il}}{r^3}\right)$$

$$= \frac{m_i (\cancel{-3 r^2 + 3 r^2}) + 3 (\vec{r} \cdot \vec{m}) \vec{r} - m r^2}{r^5} \quad \checkmark$$

Leiterschleife

$$d^3r = A\, d\vec{r}$$

$$\vec{m} = \frac{1}{2c}\int_V \vec{r}\,' \times \vec{j}(\vec{r}\,')\, d^3r'$$

$$\vec{j}\, d^3r' = I\, d\vec{r}\,'$$

$$= \frac{I}{2c}\oint_{\partial F} \vec{r}\,' \times d\vec{r}\,'$$

↓

Strom entlg d Schleife $\int$

$$j = \frac{I}{A} \qquad \text{Vol } \int \text{ zu Weg } \int$$

$$\oint_{\partial F} dx_k (\ldots) = \int_F d^2 f_i\, \varepsilon_{ijk}\, \partial_j$$

$$\oint_{\partial F} dx_k\, \varepsilon_{lmk} r'_m = \int_F d^2 f_i\, \varepsilon_{ijk} \partial'_j \varepsilon_{lmk} r'_m$$

$$= \int_F d^2 f_i\, \varepsilon_{ijk}\, \delta_{jm}\, \varepsilon_{lmk}$$

$$\int_F d^2 f_i\, \varepsilon_{imk}\, \varepsilon_{lmk} = \int_F d^2 f_i\, 2\delta_{il}$$

$$= 2\int_F d^2 f_l \quad ✓$$

$$\vec{m} = \frac{I}{c}\int_F d^2 f_l$$

Mitteln d. $\vec{B}$-Feldes // in Kugel mit Rad. R

$$\langle \vec{B} \rangle = \underbrace{\frac{3}{4\pi R^3}}_{c} \int_{\substack{V\\ r<R}} B_i(\vec{r})\, d^3r = c \int_{\substack{V\\ r<R}} \varepsilon_{ijk}\, \partial_j A_k\, d^3r$$

$$= c_1 \int_{\substack{\partial V\\ r=R}} \varepsilon_{ijk}\, d^2 f_j\, A_k = c_1 \int_{\partial V} \varepsilon_{ijk}\, d^2 f_j\, \frac{1}{c}\int_V \frac{j_k(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\, d^3r'$$

$$= c_1 \frac{1}{c}\int_V d^3r'\, \varepsilon_{ijk}\, j_k(\vec{r}\,') \int_{\partial V} d^2 f_j\, \frac{1}{|\vec{r}-\vec{r}\,'|}$$

$$= -\frac{1}{c}\int_V d^3r' \, \varepsilon_{ikj} \, j_k(\vec r') \underbrace{\frac{3}{4\pi R^3}\int_{\partial V} d^2 f_j \, \frac{1}{|\vec r - \vec r'|}}$$

$R < r'$: $\quad \dfrac{\vec r'}{r'^3}$ $\qquad\qquad$ $R > r'$: $\quad \dfrac{\vec r'}{R^3}$

$$\Rightarrow \quad \langle \vec B \rangle = -\frac{1}{c}\int_{r'>R} d^3r' \, \varepsilon_{ikj} \, j_k \, \frac{\vec r'}{r'^3} - \frac{1}{c}\int_{r'<R} d^3r' \, \varepsilon_{ikj} \, j_k \, \frac{\vec r'}{R^3}$$

$$= \frac{1}{c}\int_{r'>R} d^3r' \, \frac{\vec r' \times \vec j}{r'^3} + \frac{2}{R^3} \underbrace{\frac{1}{2c}\int_V^{r'<R} d^3r' \, \vec r' \times \vec j}_{\vec m}$$

$$\vec E = \frac{3(\vec p \, \vec r)\vec r - r^2 \vec p}{r^5} \qquad \text{Dipolfeld}$$

$$\vec B = \frac{3(\vec m \, \vec r)\vec r - r^2 \vec m}{r^5}$$

Ausrechnen – es fehlt $\qquad$ 1. Term $= \emptyset$, außer bei Strom / Ladung

$$\int_{r'<R} \vec B = \frac{4\pi \cancel{R^3}}{3} \, \frac{2\vec m}{\cancel{R^3}} = \frac{8\pi \vec m}{3}$$

$$\rightarrow \quad \vec B = \vec 0 + \frac{8\pi \vec m}{3}\,\delta(\vec r) \quad \checkmark$$

bei $\vec E$: $\quad \displaystyle\int \vec E = \frac{4\pi \cancel{R^3}}{3} \left(- \frac{\vec p}{\cancel{R^3}}\right)$

$$\rightarrow \quad -\frac{4\pi}{3}\vec p \, \delta(\vec r)$$

Unterschied nur im Kontaktterm

Magn. Multipolentw.

$j(\vec{r})$ innerhalb R

außen: $\text{rot}\,\vec{B} = \emptyset$ $\qquad |\vec{r}| > R$

$\longrightarrow \vec{B} = -\partial_i \phi_M \qquad (\text{außen})$

$\partial_i B_i = \emptyset \longrightarrow \Delta \phi_M = \emptyset$

ϕ_M bestimmt wenn am Rand ϕ_M od $\frac{\partial \phi_M}{\partial n}$ vorgegeben

$$r_i B_i = -r_i \partial_i \phi_M = r \frac{\partial \phi}{\partial n}$$

$\vec{e}_n = -\vec{e}_r \qquad // \quad \frac{\partial \phi}{\partial n} = n_i \partial_i \phi$

$$\partial_j \partial_j (r_i B_i) = r_i \partial_j \partial_j B_i = -r_i \varepsilon_{ijk} \partial_j j_k \frac{4\pi}{c}$$

$\text{rot}\,\vec{B} = \frac{4\pi}{c} \vec{j}$ $\qquad \partial_i \varepsilon_{ijk} r_j j_k \frac{4\pi}{c}$ ✓ — 2. Term mit $\delta \cdot \varepsilon = \emptyset$

$\text{grad}\,\text{div}\,\vec{B} - \Delta \vec{B} = \frac{4\pi}{c} \text{rot}\,\vec{j} \qquad \frac{4\pi}{c} \nabla (\vec{r} \times \vec{j}) = \Delta (\vec{r} \cdot \vec{B})$

$$\longrightarrow r_i B_i = -\frac{1}{c} \int_V d^3 r' \frac{\partial'_i \varepsilon_{ijk} r'_j j_k}{|\vec{r} - \vec{r}'|}$$

$$r_i \varepsilon_{ijk} \partial_j j_k = \partial_j (r_i \varepsilon_{ijk} j_k) - \underbrace{\varepsilon_{ijk} j_k \underbrace{\partial_j r_i}_{\delta_{ji}}}_{=\emptyset}$$ ✓

$$\partial_j \partial_j (r_i B_i) = \partial_j (\underbrace{\delta_{ji} B_i}_{B_j} + r_i \partial_j B_i)$$

$$= \underbrace{\partial_j B_j}_{\emptyset} + \partial_j r_i \partial_j B_i = \underbrace{\delta_{ji} \partial_j B_i}_{\emptyset} + r_i \partial_j \partial_j B_i$$ ✓

$$r_i B_i = -\frac{1}{c}\int d^3r' \; \frac{\partial_i' \, \varepsilon_{ijk} \, r_j' \, j_k}{|\vec{r}-\vec{r}'|}$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y^*_{lm}(\vartheta_<,\varphi_<)\, Y_{lm}(\vartheta_>,\varphi_>)\, \frac{r_<^{\,l}}{r_>^{\,l+1}}\, \frac{4\pi}{2l+1}$$

$$r_i B_i = -\sum_{lm} \frac{Y_{lm}(\vartheta,\varphi)}{r^{l+1}} \frac{1}{c}\int d^3r' \; \partial_i' \, \varepsilon_{ijk}\, r_j'\, j_k \quad r'^{\,l}\; Y^*_{lm}(\vartheta',\varphi')\; \frac{4\pi}{2l+1}$$

$r > r'$

$$\partial_r \frac{1}{r^{l+1}} = \partial_r \; r^{-(l+1)} = -(l+1)\; r^{-(l+1)-1}$$

$$\rightarrow \quad r^{-(l+1)} = -r\,\partial_r \frac{1}{r^{l+1}}\; \frac{1}{l+1} \quad \checkmark$$

$$\rightarrow \quad r_i B_i = +\sum_{lm} r\,\partial_r \frac{1}{r^{l+1}}\,\frac{1}{l+1}\, Y_{lm}\, \frac{1}{c}\int d^3r'\; \partial_i'\, \varepsilon_{ijk}\, r_j'\, j_k \quad r'^{\,l}\; Y^*_{lm}\; \frac{4\pi}{2l+1}$$

$$r_i B_i = -r\,\frac{\partial \phi}{\partial r}$$

$$\rightarrow \quad \phi_M = -\sum_{lm} \frac{4\pi}{2l+1}\,\frac{1}{r^{l+1}}\,\frac{1}{l+1}\; Y_{lm}\; \frac{1}{c}\int d^3r'\; \partial_i'\, \varepsilon_{ijk}\, r_j'\, j_k\, r'^{\,l}\, Y^*_{lm}$$

$$M^*_{lm} = -\frac{1}{(l+1)c}\int d^3r'\; r'^{\,l}\; Y^*_{lm}\, \partial_i'\, \varepsilon_{ijk}\; r_j'\, j_k$$

$$\rightarrow \quad M^*_{lm} = -\frac{1}{(l+1)c}\int d^3r'\; \varepsilon_{ijk}\, r_j'\, j_k\; \partial_i'\left(r'^{\,l}\, Y^*_{lm}\right)$$

// Gauß

Magnetostatische Energie

$$W = \frac{1}{8\pi} \int_V \vec{B}^2 \, d^3r = c \int B_i \, \varepsilon_{ijk} \partial_j A_k$$

$$\partial_j (B_i \varepsilon_{ijk} A_k) - \varepsilon_{ijk} A_k \partial_j B_i$$

$$— '' — \quad + \varepsilon_{kji} A_k \partial_j B_i$$

$$\text{rot } \vec{B} = \frac{4\pi}{c} \vec{j}$$

Oberflächen $\int$

$$\to \partial_j (B_i \varepsilon_{ijk} A_k) + A_k \frac{4\pi}{c} j_k$$

$$\to W = \frac{1}{2c} \int_V A_k j_k \, d^3r$$ allg. Lsg:

$$\Delta \vec{A} = -\frac{4\pi}{c} \vec{j} \quad \to \quad \vec{A}(\vec{r}) = \frac{1}{c} \int \frac{j(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3r'$$

$$\to W = \frac{1}{2c^2} \iint \frac{j(\vec{r}) \, j(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3r \, d^3r'$$

$$j(\vec{r}) = \sum_n j_n(\vec{r})$$

$$\to W = \frac{1}{2c^2} \iint \sum_n \frac{j_n \, j_n}{|\vec{r} - \vec{r}\,'|} d \ldots + \frac{1}{c^2} \iint \sum_{n<m} \frac{j_n \, j_m}{|\vec{r} - \vec{r}\,'|} d \ldots$$

✓

Induktionskoeff

mehrere Stromkreise $\quad j_n d^3r' \to I_n d\vec{r}$

$$\to W = \frac{1}{2c^2} \iint \sum_{n,m} \frac{j_n(\vec{r}) j_m(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3r' \, d^3r$$

$$\to \frac{1}{2c^2} \oint_{C_n} \oint_{C_m} \sum_{n,m} \frac{I_n I_m}{|\vec{r}_n - \vec{r}_m|} d\vec{r}_n \, d\vec{r}_m$$

$$L_{nm} = \frac{1}{c^2} \oint\oint \frac{d\vec{r}_n \, d\vec{r}_m}{|\vec{r}_n - \vec{r}_m|}$$ // Strom nicht drinnen!

$$W = \frac{1}{2} \sum_{n,m} L_{nm} I_n I_m$$

für Selbstinduktion nicht verwendbar!

Magn. Fluß d. Stromschleife

$$\phi_n = \oint_{F_n} B_i \, d^2 f_{i_n} = \oint \varepsilon_{ijk} \partial_j A_k \, d^2 f_i = \oint \varepsilon_{ijk} \partial_j \, d^2 f_{i_k}$$

$$\frac{1}{c} \int_V \frac{j_k(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3 r'$$

$$= \frac{1}{c} \oint_{C_n} \sum_{m} \int_{C_m} \frac{I_m}{|r_n - r_m|} \, d\vec{r}_n \, d\vec{r}_m$$

$$\frac{\phi_n}{c} = \sum_n I_m \underbrace{\frac{1}{c^2} \oint_{C_n}\oint_{C_m} \frac{d\vec{r}_n \, d\vec{r}_m}{|r_n - r_m|}}_{L_{nm}} \rightarrow \phi_n = c \sum_m I_m L_{nm} \checkmark$$

Lokalisierte SV im $\vec{B}$-Feld (äußeren)

$$j(\vec{r} - \vec{R})$$

$$W^{ww}(\vec{R}) = \frac{1}{c} \int_V A_k(\vec{r}) \, j_k(\vec{r} - \vec{R}) \, d^3 r$$

$$= \frac{1}{c} \int_V A_k(\vec{R} + \vec{r}) \, j_k(\vec{r}) \, d^3 r$$

$$A(\vec{R} + \vec{r}) = A(\vec{R}) + \underbrace{(\vec{r}\,\vec{\nabla})}_{r_i \partial_i A_j} \vec{A}(\vec{R})$$

$$\Rightarrow \frac{1}{c} \underbrace{\int j_k(\vec{r}) \, d^3\vec{r}}_{0} \, A(\vec{R})_k + \frac{1}{c} \int j_k(\vec{r}) \, r_i \partial_i A_k(\vec{R}) + \ldots$$

$$\vec{j}\,(\vec{r}\cdot\nabla) = \frac{1}{2}\,\vec{r}\times\vec{j}\times\vec{\nabla} + \underbrace{(\vec{\nabla}\cdot\vec{j})\,\vec{r}\;(\vec{r}\cdot\vec{\nabla})}_{\varnothing}$$

$$\to W = \frac{1}{2c}\int\left[\vec{r}\times j\times\vec{\nabla}\right]\cdot\vec{A}(\vec{R})$$

$$= \underbrace{\frac{1}{2c}\int(\vec{r}\times\vec{j})}_{\vec{m}}\cdot\underbrace{(\nabla\times\vec{A}(\vec{R}))}_{\vec{B}}\,d^3r$$

$$\varepsilon_{ijk}\,r_j\,j_k\,\varepsilon_{sim}\,\partial_m\,A_s$$

$$\text{—''—}\qquad \varepsilon_{ims}\,\partial_m\,A_s \quad \checkmark$$

$$W(R) = \vec{m}\,\vec{B}(\vec{R}) \quad \checkmark$$

Kräfte in $\vec{B}$ – Feldern

$$\vec{F} = \int_V \frac{\vec{j}}{c}\times\vec{B}\,d^3r = \frac{1}{c}\int_V \vec{j}\,(\vec{r}-\vec{R})\times B(\vec{r})\,d^3r$$

$$= \frac{1}{c}\int\vec{j}\,(\vec{r})\times\vec{B}\,(\vec{R}+\vec{r})\,d^3r$$

$$\vec{B}\,(\vec{R}+\vec{r}) \to \vec{B}(\vec{R}) + (\vec{r}\,\nabla_R)\,B(\vec{R}) + \ldots$$

$$\to \frac{1}{c}\underbrace{\int\vec{j}\,(\vec{r})}_{\varnothing}\times\vec{B}(\vec{R}) + \underbrace{\frac{1}{c}\int_V \vec{j}\,(\vec{r})\times(\vec{r}\,\nabla_R)\,B(\vec{R})\,d^3r}_{\frac{1}{c}\int_V j\,(\vec{r})\,(\vec{r}\,\vec{\nabla}_R)\times B(\vec{R})}$$

$$\vec{j}\,(\vec{r}\cdot\vec{\nabla}) = \frac{1}{2}\,\vec{r}\times\vec{j}\times\vec{\nabla} + \frac{1}{2}\underbrace{(\vec{\nabla}\cdot\vec{j})\,\vec{r}\,(\vec{r}\cdot\vec{\nabla})}_{\varnothing}$$

$$\to \vec{F}(\vec{R}) = \frac{1}{2c}\int_V\left[\vec{r}\times\vec{j}\times\vec{\nabla}\right]\times\vec{B}(\vec{R})\,d^3r$$

$$= (\vec{m}\times\vec{\nabla})\times\vec{B}(\vec{R})$$

$$\varepsilon_{ijk} m_j \partial_k \varepsilon_{lin} B_n = (\delta_{jn}\delta_{kl} - \delta_{jl}\delta_{kn}) m_j \partial_k B_n$$

$\varepsilon_{ijk}\,\varepsilon_{inl}$

$$= m_n \partial_l B_n - m_k \underbrace{\partial_n B_n}_{\emptyset}$$

$$\longrightarrow \vec{F}(\vec{R}) = \vec{\nabla}(\vec{m}\cdot\vec{B}) + \dots$$

$$W = -\int \vec{F}\, d\vec{s} = -\vec{m}\vec{B}$$

<u>Drehmoment:</u>

$$\vec{F}(\vec{r}) = \frac{1}{c}\int \vec{j}(\vec{r}) \times \vec{B}(\vec{r})\, d^3r$$

$$\vec{N}(\vec{R}) = \frac{1}{c}\int \vec{r} \times \vec{j}(\vec{r}) \times \vec{B}(\vec{R}+\vec{r})\, d^3r$$

$$\vec{B}(\vec{R}+\vec{r}) = \vec{B}(\vec{R}) + (\vec{r}\cdot\vec{\nabla}_R)\vec{B}(\vec{R}) + \dots$$

$$\rightarrow \frac{1}{c}\int \vec{r}\times\vec{j} \times \vec{B}(\vec{R}) + \underbrace{\frac{1}{c}\int \vec{r}\times\vec{j} \times (\vec{r}\,\nabla_R)\vec{B}(\vec{R})}_{\text{quadrupol}}$$

$$\vec{r}\times\vec{j}\times\vec{B} = (\vec{r}\cdot\vec{B})\vec{j} - (\vec{r}\cdot\vec{j})\vec{B}$$

$$\vec{N}(\vec{R}) = \frac{1}{c}\int_V (\vec{r}\cdot\vec{B}(\vec{R}))\vec{j} - \frac{1}{c}\int_V (\vec{r}\cdot\vec{j})\vec{B}(\vec{R})$$

$$\partial_i \underbrace{r_k r_n j_i}_{\emptyset \text{ // Oberfl}} = 2\delta_{ik} r_k j_i + r_k r_k \underbrace{\partial_i j_i}_{\emptyset \text{ // Kontgl.}}$$

$$(\vec{r}\cdot\vec{B})\vec{j} = \frac{1}{2}\vec{r}\times\vec{j}\times\vec{B} + \underbrace{\frac{1}{2}(\vec{\nabla}\cdot\vec{j})\,\vec{r}\,(\vec{r}\cdot\vec{B})}_{\emptyset}$$

$$\rightarrow \vec{N}(\vec{R}) = \frac{1}{2c}\int_V \vec{r}\times\vec{j}\times\vec{B}(\vec{R}) + \dots$$

$$= \vec{m}\times\vec{B} \quad \checkmark$$

Bewegung eines geladenen Teilchens im Feld

$$\vec{F} = \frac{q}{c}\,\vec{v} \times \vec{B}$$

$$\ddot{\vec{x}} = \frac{q}{mc}\,\vec{v} \times \vec{B}$$

Fall: hom. $\vec{B}$-Feld $\vec{B}_0$

$$\vec{\omega}_B = \frac{q}{mc}\,\vec{B}_0$$

$$\ddot{\vec{x}} = \dot{\vec{x}} \times \vec{\omega}_B$$ $\vec{B}_0$ in z-Richtung

$$\dot{\vec{v}} = \vec{v} \times \vec{\omega}_B$$ $\rightarrow \vec{\omega}_B$ auch

$$\frac{d}{dt}\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \omega_B \end{pmatrix} = \begin{pmatrix} v_y \omega_B \\ -v_x \omega_B \\ \emptyset \end{pmatrix}$$

$$\dot{v}_x = v_y\,\omega_B \qquad \ddot{v}_x = \dot{v}_y\,\omega_B$$

$$\dot{v}_y = -v_x\,\omega_B \qquad \ddot{v}_x = -v_x\,\omega_B^2$$

$$v_x = e^{\lambda t} \qquad \lambda = \pm i\,\omega_B$$

$$v_x = A\,e^{i\omega_B t} + B\,e^{-i\omega_B t}$$

$$= A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t)$$

$$= (A+B)\cos\omega t + i(A-B)\sin\omega t$$

$$v_x(\emptyset) = A + B = v_{x0} \qquad \dot{v}(\emptyset) = \emptyset \rightarrow (A-B) = \emptyset$$

$$v_x = v_{x0}\cos\omega t$$

$$v_y = \frac{\dot{v}_x}{\omega} = -v_{x0}\sin\omega t$$

✓

$$v_z = v_{z0}$$

$$\vec{v}(t) = \begin{pmatrix} v_{x0}\cos\omega t \\ -v_{x0}\sin\omega t \\ v_{z0} \end{pmatrix} \rightarrow \vec{r}(t) = \begin{pmatrix} \frac{v_{x0}}{\omega}\sin\omega t + C \\ \frac{v_{x0}}{\omega}\cos\omega t + D \\ v_{z0}t + E \end{pmatrix}$$

$$\vec{r}(0) = \begin{pmatrix} C \\ A + D \\ E \end{pmatrix} \longrightarrow \vec{r}(t) = \vec{r}(0) + \begin{pmatrix} a \sin \omega t \\ a(\cos \omega t - 1) \\ v_{z0} t \end{pmatrix}$$ ✓

Magnetische Flasche

$$\vec{B}(\rho, \varphi, z) = B_\rho(\rho, z)\, \vec{e}_\rho + B_z(z)\, \vec{e}_z$$

$$\partial_i B_i = 0 \qquad \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho B_\rho) + \partial_z B_z = 0$$

$$\partial_\rho (\rho B_\rho) = -\rho\, \partial_z B_z$$

$$\rho B_\rho = -\frac{\rho^2}{2} \partial_z B_z$$

$$\rightarrow B_\rho = -\frac{\rho}{2} \partial_z B_z$$

$$\frac{d}{dt}(v_\rho \vec{e}_\rho + v_\varphi \vec{e}_\varphi + v_z \vec{e}_z) = \frac{q}{mc} \begin{pmatrix} v_\rho \\ v_\varphi \\ v_z \end{pmatrix} \times \begin{pmatrix} B_\rho \\ 0 \\ B_z \end{pmatrix}$$

$$= \frac{q}{mc}\left(v_\varphi B_z\, \vec{e}_\rho + (v_z B_\rho - v_\rho B_z)\, \vec{e}_\varphi - v_\varphi B_\rho\, \vec{e}_z\right)$$

Näherung: $v_\varphi \approx -\omega_B \rho = -\frac{q}{mc} B_z \rho \quad // \quad \frac{v_\varphi}{B_z \rho} = -\frac{q}{mc}$

$$\frac{d}{dt} v_z = -\frac{q}{mc} v_\varphi B_\rho = \frac{v_\varphi^2 B_\rho}{B_z \rho}$$ B_ρ einsetzen!

$$\frac{d^2}{dt^2} z(t) = -\frac{v_\varphi^2}{B_z \cancel{\rho}} \frac{\cancel{\rho}}{2} \partial_z B_z = -\frac{v_\varphi^2}{2 B_z} \partial_z B_z$$ ✓

5) Wellen im Vakuum

MWGL:
$$\partial_i B_i = \emptyset \qquad \varepsilon_{ijk}\partial_j E_k = -\frac{1}{c}\partial_t B_i$$
$$\partial_i E_i = \emptyset \qquad \varepsilon_{ijk}\partial_j B_k = \frac{1}{c}\partial_t E_i$$

$$\text{rot rot } \vec{E} = (\text{grad div} - \Delta)\vec{E}$$

Term wird $\emptyset$

$$\varepsilon_{ijk}\partial_j \varepsilon_{klm}\partial_l E_m = -\frac{1}{c}\partial_t \varepsilon_{ijk}\partial_j B_k$$

$$(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\partial_j\partial_l E_m$$

$$(\underbrace{\partial_m\partial_i E_m}_{\substack{\partial_i\partial_m E_m \\ \emptyset}} - \partial_l\partial_l E_i) = -\frac{1}{c}\partial_t \frac{1}{c}\partial_t E_i$$

$$\rightarrow \quad -\partial_l\partial_l E_i + \frac{1}{c^2}\partial_t^2 E_i = \emptyset$$

$$\rightarrow \quad \Box\vec{E} = \emptyset \quad \& \quad \Box\vec{B} = \emptyset$$

Lsg mit $\vec{E}(\vec{r},t) = \vec{E}_0 \cdot f(\vec{n}\cdot\vec{r} - ct)$

Beweis: $\Delta\vec{E} = \vec{E}_0 \cdot f'' \, \vec{n}^2$ — 1

$$-\frac{1}{c^2}\partial_t^2\vec{E} = -\frac{1}{\cancel{c^2}}\vec{E}_0 f'' \cancel{c^2} \quad \checkmark$$

$$\partial_i E_i = \emptyset \quad \longrightarrow \quad n_i E_{0i} = \emptyset \quad \text{Transversal}$$

$$\varepsilon_{ijk}\partial_j E_k = -\frac{1}{c}\partial_t B_i$$

$$\varepsilon_{ijk}\partial_j E_{0k} f(n_l r_l - ct) = -\text{''}-$$

$$\varepsilon_{ijk} E_{0k} f' n_j = -\frac{1}{c}\partial_t B_i$$

$$\varepsilon_{ijk} E_{0k} f' n_j = -\frac{1}{c} \partial_t B_{0i} f(n_l r_l - ct)$$

$$\varepsilon_{ijk} E_{0k} \cancel{f'} n_j = B_{0i} \cancel{f'}$$

$$\vec{n} \times \vec{E}_0 = \vec{B}_0 \quad \checkmark$$

Monochromatische

komplex

$$\vec{E}(\vec{r},t) = \frac{1}{2}\vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)} + c.c. = \mathrm{Re}\, \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \frac{1}{2}\vec{B}_0 e^{i(\vec{k}\vec{r}-\omega t)} + c.c$$

// nur eine Frequenz bei Fourierzerlegung

$$\partial_i E_i = \emptyset \longrightarrow \frac{1}{2} i \vec{k}\, \vec{E}_0 e^{i(..)} - \frac{1}{2} i \vec{k} \vec{E}_0^* e^{-i(..)} = \emptyset$$

$$i\vec{k}\left(\frac{1}{2}\vec{E}_0(--) - \frac{1}{2}\vec{E}_0^*(--)\right) = \emptyset$$

$$\vec{k}\,\vec{E}_0 \;\&\; \vec{k}\,\vec{E}_0^* = \emptyset$$

$$\varepsilon_{ijk}\partial_j E_k = -\frac{1}{c}\partial_t B_i \qquad \text{// nur linken T nehmen}$$

$$\cancel{\frac{1}{2}}\cancel{i}\vec{k}\times\vec{E}_0 \cancel{e^{i(--)}} = +\cancel{\frac{1}{2}}\frac{1}{c}\cancel{i}\omega\,\vec{B}\cancel{e^{i(--)}}$$

oder Koeff Vgl. $\vec{k}\times\vec{E}_0 = \frac{\omega}{c}\vec{B}_0$ ✓

analog f. $\vec{B}$

Dispersionrel. aus rot MWG kann

$$\vec{k}\times\vec{k}\times\vec{E}_0 = \vec{k}\times\underbrace{\frac{\omega}{c}\vec{B}_0} \qquad -\frac{\omega^2}{c^2}\vec{E}_0$$

$$\varepsilon_{ijk} k_j \varepsilon_{klm} k_l E_{0m} = -\frac{\omega^2}{c^2}E_{0i}$$

$$(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) k_j k_l E_{0m} = -\frac{\omega}{c} E_{0i}$$

$$(k_m k_i E_{0m} - k_l k_l E_{0i}) = -\frac{\omega}{c} E_{0i}$$

$$k_m E_{0m} = \emptyset$$

$$-k_l k_l E_{0i} = -\frac{\omega^2}{c^2} E_{0i} \longrightarrow \omega^2 = \vec{k}^2 c^2 \rightarrow \omega = c|\vec{k}|$$

Polarisation

$2 e V$ senkrecht zu $\vec{k}$

$\vec{e}_i \vec{e}_j = \delta_{ij} \qquad \vec{e}_i \vec{k} = \emptyset \qquad \vec{e}_{(1)} \times \vec{e}_{(2)} = \frac{\vec{k}}{k}$

$E_1 = \vec{E}_0 \vec{e}_1 \quad \& \ E_2 \ \ldots \qquad \vec{E}_0$ komplex $\rightarrow$ E_1 komplex.

$$\vec{E} = \frac{1}{2}\left(|E_1| e^{i\varphi_1} \vec{e}_1 + |E_2| e^{i\varphi_2} \vec{e}_2\right) e^{i(\vec{k}\vec{r} - \omega t)} + c.c.$$
$$= \frac{1}{2}\left(|E_1| \vec{e}_1 + |E_2| e^{i(\varphi_2 - \varphi_1)} \vec{e}_2\right) e^{i(\vec{k}\vec{r} - \omega t)} e^{i\varphi_1} + c.c.$$

Für Polari. nur Beträge & Phasendiff wichtig

zirkular pol.:

$\vec{e}_+ = \frac{1}{\sqrt{2}}(\vec{e}_1 + i \vec{e}_2) \qquad E_+ = \vec{E}_0 \vec{e}_+^{\,*}$

$\vec{e}_- = \frac{1}{\sqrt{2}}(\vec{e}_1 - i \vec{e}_2) \qquad E_-$ analog

$\vec{e}_\pm \vec{e}_\pm^{\,*} = \frac{1}{2} 1 + \frac{1}{2} 1 = 1$

$\vec{e}_\pm \vec{e}_\mp^{\,*} = \frac{1}{2} 1 - \frac{1}{2} 1 = \emptyset \qquad \vec{e}_\pm \vec{k} = \emptyset$

$\rightarrow \vec{E}(\vec{r}, t) = \frac{1}{2}(E_+ \vec{e}_+ + E_- \vec{e}_-) e^{i(\vec{k}\vec{r} - \omega t)} + c.c.$

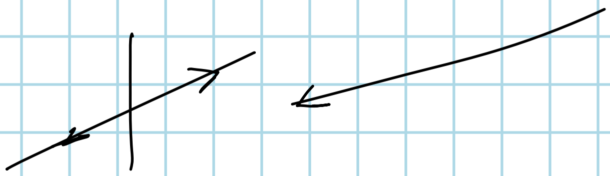
$E_+ = \vec{E}_0 \vec{e}_+^{\,*} = (E_1 \vec{e}_1 + E_2 \vec{e}_2) \cdot \frac{1}{\sqrt{2}}(\vec{e}_1 - i \vec{e}_2)$
$= \frac{1}{\sqrt{2}}(E_1 - i E_2)$ ✓

$E_- = \frac{1}{\sqrt{2}}(E_1 + i E_2)$

$\rightarrow E_1 = (E_+ + E_-) \underbrace{\frac{\sqrt{2}}{2}}$

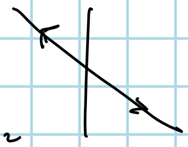
$\frac{\cancel{\sqrt{2}\sqrt{2}}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$ ✓

4 Mög. f. Phasenversch. $\Delta\varphi = \varphi_2 - \varphi_1 = 0$

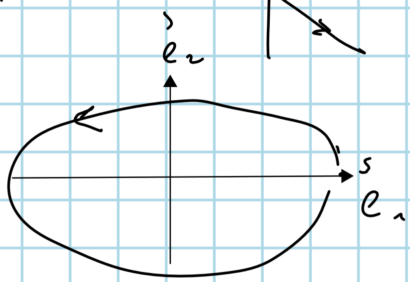


$\Delta\varphi$ ist bei $|E_2|$ gegenüber $|E_1|$!

$\Delta\varphi = \pi \hat{=} -\pi$

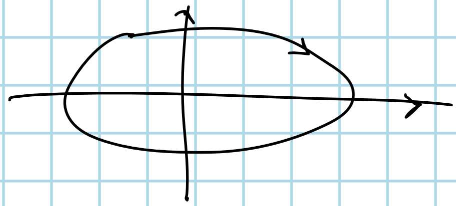


$\Delta\varphi = \frac{\pi}{2}$



zuerst E_1 voll dann E_2

$\Delta\varphi = -\frac{\pi}{2}$



hier ist E_2 vorn

$$\vec{E}_+ = \frac{1}{2} \overset{\wedge}{E_+} \vec{e}_+ e^{i(\vec{k}\vec{r}-\omega t)} + \frac{1}{2} \overset{\wedge}{E_+^*} \vec{e}_+^{\,*} e^{-i(\vec{k}\vec{r}-\omega t)}$$

$$= \frac{1}{2}\frac{1}{\sqrt{2}}(\vec{e}_1 + i\vec{e}_2)\, e^{i(\ldots)} + \frac{1}{2}\frac{1}{\sqrt{2}}(\vec{e}_1 - i\vec{e}_2)\, e^{-i(\ldots)}$$

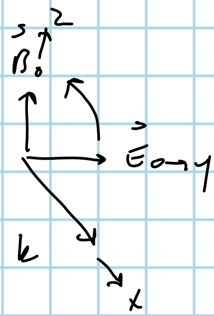
$\vec{e}_1$ & $\vec{e}_2$ rausheben

$$= \frac{1}{\sqrt{2}}\vec{e}_1 \cos(\vec{k}\vec{r}-\omega t) - \frac{1}{\sqrt{2}}\vec{e}_2 \sin(\vec{k}\vec{r}-\omega t)$$

mit ↗

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} - e^{-ix}}{2}$$



$$\sin(-x) = -\sin(x)$$

$$\rightarrow +\frac{1}{\sqrt{2}} e_2 (\omega t - \vec{k}\vec{r})$$ // sin wird ↑ mit steig. t

cos ↓

Energie & Impuls v. ebenen Wellen (monochr.)

Pointing - V. $\quad \vec{S} = \frac{c}{4\pi}(\vec{E}\times\vec{B})^{(\vec{r},t)}$

$$\vec{E}(\vec{r},t) = \tfrac{1}{2}\vec{E}_0\, e^{i(\vec{k}\vec{r}-\omega t)} + c.c.$$

$$S_i = \frac{c}{4\pi}\varepsilon_{ijk} E_j B_k = \frac{c}{4\pi}\varepsilon_{ijk}\frac{1}{4}\left(E_{0j}\, e^{i(k_l r_l-\omega t)} + E_{0j}^{*}\, e^{-i(-\!-\!-)}\right)$$

$$\left(B_{0k}\, e^{i(-\!-\!-)} + B_{0k}^{*}\, e^{-i(-\!-\!-)}\right)$$

$$(a+a^*)(b+b^*) = ab + a^+b + ab^* + a^+b^+$$

$$(ab + ab^+) + c.c.$$

$$\rightarrow\ S_i = \frac{c}{4\pi}\varepsilon_{ijk}\frac{1}{4}\left(E_{0j} B_{0k}\, e^{2i(-\!-\!-)} + E_{0j} B_{0k}^{+}\right) + c.c.$$

$$B_{0k} = \varepsilon_{klm}\, n_l E_{0m}$$

$$\varepsilon_{ijk} E_{0j}\, \varepsilon_{klm}\, n_l E_{0m} = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) -\!-\!-$$

$$E_{0m} n_i E_{0m} - \underbrace{E_{0l} n_l}_{\emptyset} E_{0i}$$

$$\longrightarrow\ S_i = \frac{c}{4\pi}\, n_i\, \frac{1}{4}\left(E_{0m}E_{0m}^{*} + E_{0m}E_{0m}\, e^{2i(-\!-\!-)}\right) + c.c$$

$$w(\vec{r},t) = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{B}^2\right)$$

$$= \frac{1}{8\pi}\Big[\left(\tfrac{1}{2}E_{0j}\, e^{i(-\!-\!-)} + \tfrac{1}{2}E_{0j}^{+}\, e^{-i(-\!-)}\right)\left(\tfrac{1}{2}E_{0j}\, e^{i(-\!-\!-)} + \tfrac{1}{2}E_{0j}^{+}\, e^{-i(-\!-\!-)}\right)$$

$$+ \ B - \text{Anteil}\Big]$$

$$= \frac{1}{8\pi}\frac{1}{4}\Big[E_{0j}E_{0j}\, e^{2i(-\!-\!-)} + 2E_{0j}E_{0j}^{+} + E_{0j}^{+}E_{0j}^{+}\, e^{-2i(-\!-\!-)} + B\ \text{Teil}\Big]$$

$$//\ (a+a^+)(a+a^+) = a^2 + 2aa^+ + a^{+2}$$

$$= \frac{1}{8\pi}\frac{1}{4}\Big[E_{0j}E_{0j}\, e^{2i(-\!-\!-)} + E_{0j}E_{0j}^{+} + B\ \text{Anteil}\Big] + c.c.$$

$$B_i B_i = \varepsilon_{ijk}\, n_j E_k\, \varepsilon_{ilm}\, n_l E_m$$

$$(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}) -\!-\!- = n_l E_m n_l E_m$$

$$- n_m \underbrace{E_l n_l}_{\emptyset} E_m$$

$$\rightarrow \frac{1}{8\pi}\frac{1}{4}\left[2 E_{0j} E_{0j}\, e^{2i(\ldots)} + 2 E_{0j} E_{0j}^{*} \right] + c.c.$$

$$\rightarrow w_{em}(\vec{r},t) = \frac{1}{4\pi}\frac{1}{4}\left[E_{0j} E_{0j}\, e^{2i(\ldots)} + E_{0j} E_{0j}^{*} \right] + c.c.$$

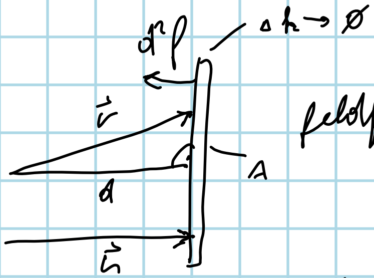
$$\vec{S} = \vec{n}\, c\, w_{em}$$

Spannungstensor & Impulsstromdichte

Kraft auf Plättchen

$$T_{ij} = \frac{1}{4\pi}\left(E_i E_j + B_i B_j \right) - \frac{1}{8\pi}\left(E_i E_i + B_i B_i \right)\delta_{ij}$$

$$\vec{F}_{mech}(t) = \frac{d}{dt}\vec{p}_{mech}(t) = \int_V \nabla \overleftrightarrow{T}\, d^3r - \frac{d}{dt}\int_V \vec{g}_{em}\, d^3r$$



feldfrei

V um Plättchen. beliebig kleines V

$$\rightarrow \nabla \cdot T \rightarrow \emptyset$$

$$\int_V \vec{\nabla}\cdot\overleftrightarrow{T}\, d^3r \longrightarrow \int_{\partial V} d^2\vec{f}\cdot\overleftrightarrow{T}$$

$$\oint_{\partial V} d^2\vec{f}\, \overleftrightarrow{T} = -\int_{\partial V} d^2 f\, \vec{n}\, \overleftrightarrow{T} = -A\, \vec{n}\, T_{nn} \Big|_{\vec{n}\vec{r} = d}$$

(nur Anteil von vorne, hinten gibt es keine Felder bei absorbierendem Plättchen)

n kennz. Ausbreitungsr. d. Welle

$\int$ nicht Ortsabh. $\vec{n}\cdot\vec{r} = \text{const}$ auf Vorderfl.

$$\vec{n}\,\overleftrightarrow{T} = \frac{1}{4\pi}\underbrace{\vec{n}\cdot\vec{E}}_{\emptyset}\, E - \vec{n}\cdot w_{em}$$

$$= -\vec{n}\, w_{em} = -c\,\vec{g}_{em}$$

$$T_{nn} = \frac{1}{4\pi}\left(E_n E_n + B_n B_n \right) - \underbrace{\frac{1}{8\pi}\left(\vec{E}^2 + \vec{B}^2 \right)}_{w_{em}}$$

$\emptyset$ f. ebene Welle

// Komp. in Richtg. d. Normalvektors

$\rightarrow$ $\vec{F}_{med}$ nur von t abh.

$$\vec{F}_{med} = A\,\vec{n}\, w_{em}|_{\vec{n}\vec{r}=d}$$ (Felder sind auf Fläche überall gleich)

Zeitabh.:

$\vec{S}$ f. lin. pol.:

$$S_i = n_i\, c\, w_{em}$$

$$= n_i \frac{c}{8\pi}\left(\vec{E}^2 + \vec{B}^2\right) = n_i \frac{c}{4\pi}\vec{E}^2$$

// f. $\vec{B}^2$ kommt d. $\vec{n}\times\vec{E}$ das gleiche

l.p.

$$E_i = \frac{1}{2}\left(|E_1|\, e_{1i} + \underbrace{|E_2|\, e_{2i}\, e^{i(\varphi_2-\varphi_1)}}_{\emptyset}\right) e^{i(k_j r_j - \omega t + \varphi_1)} + c.c.$$

weil nur 1. l.p.
// nur bei manchen.

$$\vec{E}^2 = \frac{1}{4}|E_1|^2 e^{2i(-\,-)} + \frac{1}{2}|E_1|^2 + \frac{1}{4}|E_1|^2 e^{-2i(-\,-)}$$

$$= \frac{1}{2}|E_1|^2\left(\frac{1}{2}e^{2i(-\,-)} + \frac{1}{2}e^{-2i(-\,-)} + 1\right)$$

$$= \frac{1}{2}|E_1|^2\left(\cos\left(2\left(\vec{k}\vec{r} - \omega t + \varphi_1\right)\right) + 1\right)$$

$\longrightarrow$ schwankt um MW mit dopp. Frequ.

links z. pol. Welle

$$\vec{E}(\vec{r},t) = \frac{1}{2}\left(E_+\vec{e}_+ + \underbrace{E_-\vec{e}_-}_{\emptyset}\right) e^{i(\vec{k}\vec{r} - \omega t)} + c.c.$$

$$\rightarrow \vec{E}_+ = \frac{1}{2}E_+ \frac{1}{\sqrt{2}}\left(\vec{e}_1 + i\,\vec{e}_2\right) e^{i(-\,-)} + c.c.$$

$$= |E_+|\, e^{i\varphi_1}\, \frac{1}{2}\frac{1}{\sqrt{2}}\left(-\,-\right) e^{i(-\,-)} + c.c.$$

$$= \frac{1}{\sqrt{2}}|E_+|\left(\vec{e}_1 \cos\left(\vec{k}\vec{r} - \omega t + \varphi_1\right) - \vec{e}_2 \sin\left(-\,-\right)\right)$$

weil i drin!

$\vec{E}^2 \propto \cos^2 + \sin^2 = 1$

$\rightarrow$ zirk. pol. Welle $\vec{S}$ zeitunabh. linpol. zeitabh.!