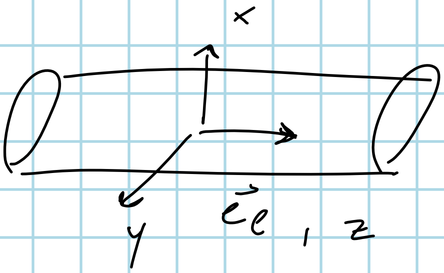
V.4 Wellen im Hohlleiter



$$\vec a = \vec a_l + \vec a_t = a_l \vec e_l + \vec a_t$$

$$a_l = \vec a \cdot \vec e_l \qquad \vec a_t = \vec a - a_l \vec e_l$$

$$= \vec e_l \times (\vec a \times \vec e_l)$$

$$\vec E_{tg}\big|_{\partial V} = 0 \qquad \vec B_n \big|_{\partial V} = 0$$

$$\longrightarrow \quad \vec E \times d^2\vec f = 0 \qquad \vec B \cdot d^2\vec f = 0$$

Zerlegung d. MWG:

$$\partial_i E_i = 0 \qquad \varepsilon_{ijk}\partial_j E_k = -\frac{1}{c}\partial_t B_i$$

$$\partial_i B_i = 0 \qquad \varepsilon_{ijk}\partial_j B_k = \frac{1}{c}\partial_t E_i$$

aufteilen $\quad \partial_i^t E_i^t + \partial_i^l E_i^l = 0$

same f. $\vec B$

$$\begin{pmatrix}\partial_x\\ \partial_y\\ 0\end{pmatrix} \times \begin{pmatrix}E_x\\ E_y\\ 0\end{pmatrix} = \begin{pmatrix}0\\ 0\\ \cdot -\end{pmatrix} \qquad \begin{aligned}\vec\nabla_t \times \vec E_t &= -\frac{1}{c}\partial_t \vec B_l\\ \vec\nabla_t \times \vec B_t &= \frac{1}{c}\partial_t \vec E_l\end{aligned}$$

$$\vec\nabla_t \times \vec E_l + \vec\nabla_l \times \vec E_t = -\frac{1}{c}\partial_t B_l$$

$$\vec\nabla_t \times \vec B_l + \vec\nabla_l \times \vec B_t = \frac{1}{c}\partial_t E_l$$

$$(\vec\nabla_l + \vec\nabla_t)(\vec E_l + \vec E_t) = 0 \qquad // \ \vec\nabla_l \vec E_t = 0$$ weil unterschiedl. Komp.

rot auf rot Gl. & div einsetzen

$$\longrightarrow \quad \Box \vec E = 0 \qquad \Box \vec B = 0$$

$$\left(\Delta_t + \Delta_l - \frac{1}{c^2}\partial_t^2\right)\vec E = 0$$ same f. $\vec B$

Long. Komp.:

$$-\Delta_t E_l = \left(\Delta_l - \frac{1}{c^2}\partial_t^2\right) E_l$$

same f. B_l

transvers.

$$\left(\Delta_l - \frac{1}{c^2}\partial_t^2\right) E_t = -\Delta_t E_t$$

$$\Delta_t \vec{E}_t = \vec{\partial}_t(\vec{\nabla}_t \cdot \vec{E}_t) - \vec{\nabla}_t \times (\vec{\nabla}_t \times \vec{E}_t)$$

$$= -\vec{\partial}_t(\vec{\nabla}_l \vec{E}_l) + \frac{1}{c}\vec{\nabla}_t \times (\partial_t \vec{B}_l)$$

oben einsetzen $\to$ fertig !

TEM – Wellen

allg. Ansatz f. Wellen in z-Richtg

$$\vec{E}(\vec{r},t) = \vec{E}(x,y)\, e^{i(kz-\omega t)} + c.c$$

same $\vec{B}$

long. Komp

$$-\Delta_t E_l = \left(\Delta_l - \frac{1}{c^2}\partial_t^2\right) E_l$$

$$\Delta_t E_l = \left(k^2 - \frac{\omega^2}{c^2}\right) E_l \qquad // \text{ same f. } \vec{B}$$

transvers. K.

$$\left(\Delta_l - \frac{1}{c^2}\partial_t^2\right)\vec{E}_t = -\Delta_t \vec{E}_t$$

$$= \vec{\nabla}_t \times (\vec{\nabla}_t \times \vec{E}_t) - \vec{\partial}_t(\vec{\nabla}_t E_t)$$

$$= \frac{1}{c}\partial_t(\nabla_t \times \vec{B}_l) + \vec{\partial}_t(\vec{\partial}_l \vec{E}_l)$$

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)\vec{E}_t = -i\frac{\omega}{c}(\nabla_t \times \vec{B}_l) + ik(\vec{\partial}_t \vec{E}_l)$$

$B_l \cdot \vec{e}_z$

$(\nabla_t B_l \times \vec{e}_z)$ same f. $\vec{B}$

TEM – Wellen : $\vec{B}_\ell$ & $\vec{E}_\ell = 0$

$\rightarrow \left(k^2 - \frac{\omega^2}{c^2}\right)\vec{E}_t = 0$

Disp Rel. $k^2 = \frac{\omega^2}{c^2}$

f. transv. Komp. $-\Delta_t \vec{E}_t = \underbrace{\left(\Delta_\ell - \frac{1}{c^2}\partial_t^2\right)}_{0}\vec{E}_t$

$\rightarrow \Delta_t \vec{E}_t = 0$ sowie f $\vec{B}$

Berücks. d. RB $E_{t\parallel} = 0$ am Rand

$\vec{E}_n \times d^2\vec{f} = 0$ ✓

außerdem $\vec{\nabla}_t \times \vec{E}_t = -\frac{1}{c}\partial_t \vec{B}_\ell = 0$ | TEM

$\rightarrow \vec{E}_t = -\vec{\nabla}_t \phi(x,y)$

Poti. erfüllt $\vec{\nabla}_t \vec{E}_t = -\vec{\nabla}_\ell \vec{E}_\ell = 0$

$\rightarrow \Delta \phi = 0$

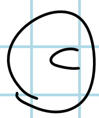
$\vec{E}_{t\parallel} = \vec{E}_\ell$

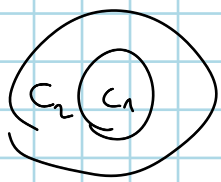
$\vec{E}_n = \vec{E}_t$

$\vec{E}_{t\parallel} = 0$ am Rand

$t\parallel$ auf Fläche

$\rightarrow \phi$ const auf Oberfläche da $\vec{E} = -\nabla\phi$

Bei nur einf. z. QS.  gibt es daher kein

Feld; bei  schon

siehe 2 Lsg bei Vorgabe eines konst Potis

TM - Wellen

$B_z = 0$

$$-\Delta_t E_z = \underbrace{\left(\Delta_\ell - \frac{1}{c^2}\partial_t^2\right)}_{\underbrace{\left(-k^2 + \frac{\omega^2}{c^2}\right)}_{\lambda^2}} E_z$$

$E_z = 0$ am Rand

$$-\Delta_t E_z^{(n)} = \lambda^{(n)2} E_z \qquad \text{Eigenwertproblem}$$

$$\lambda^{(n)2} = \frac{\omega^2}{c^2} - k^{(n)2}$$

$$k^{(n)2} = \frac{\omega^2}{c^2} - \lambda^{(n)2}$$

λ^{n2} größer $\rightarrow$ k^2 kleiner

irgendwann neg.

$\rightarrow$ k imaginär $\rightarrow$ abklingende Welle

nicht ausbreiten

E_z gelöst, E_t & B_t berechnen

TE analog wie TM

TE: $E_z = 0$

$$-\Delta_t B_z = \text{—"—} \quad B_z$$

B_z am Rand $= 0$

gleiches EW-Problem

RB für B_z

normal auf Fläche

$\vec{B}_n = \vec{B}_t$ — tang. auf dersch. nötig.

$$d\vec{f} \cdot \vec{B} = d^2\vec{f} \cdot \vec{B}_t = 0$$

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)\vec{B}_t = ik\,\vec{\nabla}_t B_z \rightarrow B_t = \frac{ik}{\frac{\omega^2}{c^2} - k^2}\vec{\nabla}_t B_z$$

$$\rightarrow \quad d^2\vec{f}\,\vec{\nabla}_t B_z = d^2 f \frac{\partial}{\partial n} B_z = 0$$

Neumann RB

VI Feld eines bewegten T. :

Bahnkurve $\vec{R}(t)$ LW - Potentiale

$$\rho = q\,\delta(\vec{r} - \vec{R}(t))$$
$$j = q\,\vec{v}(t)\,\delta(\vec{r} - \vec{R}(t)) \qquad \vec{v}(t) = \frac{d\vec{R}(t)}{dt}$$

$$\frac{\partial \rho}{\partial t} = q\,\delta(\vec{r} - \vec{R}(t)) \cdot - \frac{\partial \vec{R}}{\partial t}$$

Potentiale ausrechnen

$$\phi(\vec{r},t) = \iint d^3r'\,dt'\,\frac{1}{|\vec{r}-\vec{r}'|}\,\delta\left(t'-t+\frac{|\vec{r}-\vec{r}'|}{c}\right)\rho(\vec{r}',t')$$

$$\vec{A}(\vec{r},t) = \frac{1}{c}\iint d^3r'\,dt'\,\frac{1}{|\vec{r}-\vec{r}'|}\,\delta(-\,"\,-)\,\vec{j}(\vec{r}',t')$$

einsetzen $\longrightarrow$ $\phi(\vec{r},t) = \int dt'\,\frac{q}{|\vec{r}-\vec{R}(t')|}\,\delta\Big(\underbrace{t'-t+\frac{|\vec{r}-\vec{R}(t')|}{c}}_{f}\Big)$ ✓

$\vec{A}$ analog , t' integ.

Zusammenges. δ fkt $\rightarrow$ Regel

$$\delta(f(x)) = \frac{\delta(t' - Nst)}{|f'(Nst)|} \qquad t' = t - \frac{|r - R(t')|}{c}$$

Stelle wo dies gilt $\rightarrow$ T

$$\longrightarrow \delta\left(t'-t+\frac{|\vec{r}-\vec{R}(t')|}{c}\right) = \frac{\delta(t'-T)}{|f'(T)|}$$

$$= \frac{\delta(t'-T)}{1 - \frac{\vec{r}-\vec{R}(T)}{|\vec{r}-\vec{R}(T)|}\cdot\frac{\vec{v}(T)}{c}}$$

$$\longrightarrow \phi(\vec{r},t) = \frac{q}{|\vec{r}-\vec{R}(T)|}\;\frac{1}{1 - \frac{\vec{r}-\vec{R}(T)}{|\vec{r}-\vec{R}(T)|}\,\frac{\vec{v}(T)}{c}}$$

$\vec{A}$ analog

Einführen von Abkürzungen

$$\vec{r} - \vec{R}(\tau) = \vec{r}_\tau \qquad \vec{n}_\tau = \frac{\vec{r}_\tau}{r_\tau} \qquad \vec{\beta}_\tau = \frac{\vec{v}(\tau)}{c}$$

$$\phi(\vec{r},t) = \frac{q}{r_\tau} \frac{1}{1 - \vec{n}_\tau \vec{\beta}_\tau} \qquad \kappa = 1 - \vec{n}_\tau \vec{\beta}_\tau$$

$$= \frac{q}{r_\tau \kappa} \qquad \tau = t - \frac{r_\tau}{c}$$

$\vec{A}$ analog $\qquad \vec{A} = \phi \vec{\beta}_\tau$

Felder berechnen mit

$$\vec{B} = \operatorname{rot} \vec{A}$$

$$\vec{E} = -\operatorname{grad} \phi - \frac{1}{c} \partial_t \vec{A}$$

$$\longrightarrow \vec{E} = \frac{q}{r_\tau^2 \kappa^3} (1 - \beta_\tau^2)(\vec{n}_\tau - \vec{\beta}_\tau)$$

$$+ \frac{q}{r_\tau \kappa^3} \vec{n}_\tau \times \left[(\vec{n}_\tau - \vec{\beta}_\tau) \times \frac{1}{c} \partial_\tau \vec{\beta}_\tau \right]$$

Gleichförmig bewegt. Teilch

$$\vec{R}(t) = \vec{v} t$$

$$\phi(\vec{r},t) = \frac{q}{r_\tau \kappa} = \frac{q}{|\vec{r} - \vec{R}(\tau)|} \frac{1}{1 - \frac{\vec{r} - \vec{R}(\tau)}{|\vec{r} - \vec{R}(\tau)|} \frac{\vec{v}(\tau)}{c}}$$

$$r_\tau \kappa = r_\tau (1 - \vec{n}_\tau \vec{\beta}_\tau) = r_\tau - \vec{r}_\tau \vec{\beta}_\tau$$

retardierte Größen rausbringen

$$\vec{r}_t = \vec{r} - \vec{v} t \qquad \vec{r}_\tau = \vec{r} - \vec{v} \tau$$

aus $\tau = t - \frac{\overbrace{|\vec{r} - \vec{R}(\tau)|}^{r_\tau}}{c}$

$$\vec{r}_t = \vec{r}_\tau - v(t - \tau)$$

$$\longrightarrow r_\tau = c(t - \tau)$$

$$\longrightarrow \vec{r}_t = \vec{r}_\tau - \vec{\beta} r_\tau$$

$$(r_+ \times \beta) = (r_J \times \beta) \quad \text{weil} \quad r_+ - r_J \parallel \beta$$

$$\vec{r}_+^{\,2} - (\vec{r}_+ \times \vec{\beta})^2 = \underbrace{(\vec{r}_J - \vec{\beta} r_J)^2}_{r_J^2 - 2\vec{r}_J\vec{\beta} r_J + (\vec{\beta} r_J)^2} - (\vec{r}_J \times \vec{\beta})^2$$

$$(\vec{r}_J \times \vec{\beta})^2 = \varepsilon_{ijk} r_j \beta_k \, \varepsilon_{ilm} r_l \beta_m$$

$$(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{lk}) r_j \beta_k r_l \beta_m = r_l \beta_m r_l \beta_m - r_m \beta_l r_l \beta_m$$

$$= r_J^2 \beta^2 - (\vec{r}_J \vec{\beta})^2$$

$$\rightarrow \quad r_+^2 - (r_+ \times \vec{\beta})^2 = r_J^2 - 2\vec{r}_J \vec{\beta} r_J + (\vec{r}_J \vec{\beta})^2$$

$$= (r_J - \vec{r}_J \vec{\beta})^2$$

$$\rightarrow \quad \phi(\vec{r}, t) = \frac{q}{r_J K} = \frac{q}{r_J - \vec{r}_J \vec{\beta}} = \frac{q}{\sqrt{r_+^2 - (\vec{r}_+ \times \vec{\beta})^2}} \checkmark$$

Äquipotentialf.

wo nenner $=$ const

$$\vec{r}_+^{\,2} - (\vec{r}_+ \times \vec{\beta})^2 \quad \text{mit} \quad \vec{r}_+ = \vec{r}_\perp + \vec{r}_\parallel \quad // \text{ bzgl } \vec{\beta}$$

$$(\vec{r}_\perp + \vec{r}_\parallel)^2 - (\underbrace{(\vec{r}_\perp + \vec{r}_\parallel) \times \vec{\beta}}_{\vec{r}_\perp \times \vec{\beta} + \underbrace{\vec{r}_\parallel \times \vec{\beta}}_{\emptyset}})^2$$

$$r_\perp^2 + 2\underbrace{r_\perp r_\parallel}_{\emptyset} + r_\parallel^2 - (r_\perp^2 \beta^2 - \underbrace{(r_\perp \cdot \beta)}_{\emptyset}{}^2)$$

$$= r_\parallel^2 + r_\perp^2(1 - \beta^2) = \text{const}$$

Berechnung d $\vec{E}$ – Feldes

$$E_i = -\partial_i \phi = -\partial_i \frac{q}{r_J K} = -\partial_i \frac{q}{\sqrt{v_+^2 - (v_+ \times \beta)^2}}$$

Geschwindigkeit → const.

$$\vec{E} = \frac{q}{r_J^2 K^3} (1-\beta_J^2)(\vec{n}_J - \vec{\beta}_J) \quad \| + \frac{q}{r_J K^3} \vec{n}_J \times ((\vec{n}_J - \vec{\beta}_J) \times \frac{1}{c} \partial_J \vec{\beta})$$

$$\rightarrow \vec{E} = \frac{q}{r_J^3 K^3} (1-\beta^2)(\underbrace{\vec{r}_J - \vec{\beta} r_J}_{\vec{r}_+})$$

$$= \frac{q(1-\beta^2)}{\sqrt{r_+^2 - (r_+ \times \beta)^2}} \vec{r}_+$$

Berechng. d $\vec{B}$ – Felds

$$\vec{B} = \vec{n}_J \times \vec{E} \quad \text{von } \vec{r}_+ \qquad \vec{r}_+ = \vec{r}_J - \vec{\beta} r_J$$

$$\frac{\vec{r}_J}{r_J} = \frac{\vec{r}_+ + \vec{\beta} r_J}{r_J}$$

$$\rightarrow \vec{B} = \vec{\beta} \times \vec{E} \quad ✓$$

Kraft auf ein mitbewegtes Teilchen

$$\vec{F} = Q(\vec{E} + \vec{\beta} \times \vec{B})$$

2. T hat Ladung Q

$$\phi = \phi(\vec{r} - vt) \qquad \vec{A} = \phi \vec{\beta}$$

$$\vec{E} = -\operatorname{grad} \phi - \frac{1}{c} \partial_t A \qquad \vec{B} = \operatorname{rot} \vec{A}$$

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\partial_t(\phi\vec{\beta}) \qquad \vec{B} = \mathrm{rot}(\phi\vec{\beta})$$

$$-\vec{\nabla}\phi - \frac{1}{c}\dot{\phi}\vec{\beta} = -\nabla\phi + \vec{\beta}(\vec{\nabla}\phi\cdot\vec{\beta})$$

$$// \ \dot{\phi} = -\vec{\nabla}\phi\cdot\vec{v} \qquad \vec{B} = \vec{\nabla}\phi\times\vec{\beta}$$

$$\rightarrow \vec{F} = Q\left(-\vec{\nabla}\phi + \vec{\beta}(\vec{\nabla}\phi\cdot\vec{\beta}) + \vec{\beta}\times(\vec{\nabla}\phi\times\vec{\beta})\right)$$

$$\varepsilon_{ijk}\beta_j\,\varepsilon_{klm}\partial_l\phi\,\beta_m$$

$$(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\beta_j\partial_l\phi\,\beta_m = \beta_m\partial_i\phi\,\beta_m - \beta_l\partial_l\phi\,\beta_i$$

$$\rightarrow \vec{\beta}^2\vec{\nabla}\phi - \vec{\beta}(\vec{\beta}\cdot\vec{\nabla}\phi)$$

$$\rightarrow \vec{F} = Q\left(-\vec{\nabla}\phi + \vec{\beta}^2\vec{\nabla}\phi\right) = Q\vec{\nabla}\phi(\vec{\beta}^2 - 1) \checkmark$$

Zeigt in Richtung Flächen, d. Äquipotfl.

Bewegte Ladungen – Abst. v. Wellen $\qquad \vec{r}\to\infty$

$$\vec{A}(\vec{r},t) = \frac{1}{c}\int d^3r' \frac{\vec{j}\left(\vec{r}', t - \frac{|\vec{r}-\vec{r}'|}{c}\right)}{|\vec{r}-\vec{r}'|} \qquad // \ \mathrm{rot} \ t' \int$$

$$\vec{B} = \mathrm{rot}\,\vec{A}$$

$$\vec{B} = \frac{1}{c}\int d^3r'\left(\frac{1}{|\vec{r}-\vec{r}'|}\vec{\nabla}\times\vec{j} - \vec{j}\times\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$\vec{\nabla}\times\vec{j}: \quad \varepsilon_{ijk}\partial_j j_k = \varepsilon_{ijk}\dot{j}_k - \frac{1}{c}\frac{(r_j - r_j')}{|r_a - r_a'|}$$

$$\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|} = -\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{B} = \frac{1}{c}\int d^3r' \left(-\frac{(\vec{r}-\vec{r}')}{c|\vec{r}-\vec{r}'|^2} \times \dot{\vec{j}} + \underbrace{\vec{j} \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}_{\frac{1}{r^2}} \right) \checkmark$$

Entwicklg nach $\frac{1}{r}$

$$\vec{f}(\vec{r}) = \vec{f}(\vec{r}_0) + \partial_j \vec{f}(\vec{r}_0)(\vec{r}-\vec{r}_0)_j \qquad r \to \infty \quad r' \to \emptyset$$

$$\vec{f} = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^2} \qquad \partial_j |\vec{r}-\vec{r}'| = \frac{\vec{r}-\vec{r}'_j}{|\vec{r}-\vec{r}'|}$$

$$\sqrt{(\vec{r}-\vec{r}')^2} = (r^2 - 2rr' + r'^2)^{\frac{1}{2}}$$

// egal nach was man hier ableitet, es kommt d. selbe raus

$$\Rightarrow \frac{1}{2}(-\text{"}-)^{-\frac{1}{2}}(2\vec{r} - 2r')$$

Entw um r' entwickeln — nach r' ableiten und r' einsetzen

$$\partial_j \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^2} = (\vec{r}-\vec{r}'|_i \cdot - \frac{(\vec{r}-\vec{r}')_j}{|\vec{r}-\vec{r}'|^4} + \frac{1}{|\vec{r}-\vec{r}'|^2} - \delta_{ij}$$

$$\rightarrow \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^2} = \frac{\vec{r}}{r^2} + \left[r'_j \cdot \left(\frac{r_i r_j}{r^4} - \frac{\delta_{ij}}{r^2} \right) \right] \text{— egal}$$

nur 1. T $\sim \frac{1}{r}$ ✓

→ Strahlungsanteil d. Feldes

$$\rightarrow \vec{B} = \frac{1}{c}\int d^3r' - \frac{1}{c}\frac{\vec{r}}{r^2} \times \dot{\vec{j}}\left(\vec{r}', t - \underbrace{\frac{|r-\vec{r}'|}{c}}\right)$$

Entwickeln u Zeitarg

$|\vec{r}-\vec{r}'|$:

Ableitg egal oder einsetzen: $\underline{r = 0}$:

$$r' - \vec{r} \cdot \frac{\vec{r}'}{r'} \qquad \underline{r' = 0}: \quad r - \frac{\vec{r}' \cdot \vec{r}}{r} \Bigg\} \text{ fertige Entw}$$

$$\rightarrow \vec{B} = -\frac{1}{c^2 r^2}\int d^3r' \; \vec{r} \times \dot{\vec{j}}\left(\vec{r}', t - \frac{r}{c} + \frac{\vec{r}\cdot\vec{r}'}{r}\right)$$

$$\dot{\vec{q}} = \int d^3r' \; \vec{j}(--)$$

$$\to \quad \vec{B}_s = \frac{\ddot{\vec{q}} \times \vec{r}}{c^2 r^2} \quad \checkmark$$

Einf. Näherung v. q. el. Dipolmom.

Dipolm.: $\int d^3r \; \dot{\vec{j}}\, \vec{r}$ ---

Berechnung v. $\vec{E}_s$ $\qquad \varepsilon_{ijk} \partial_j B_k = \frac{1}{c} \partial_t E_i$

$$\frac{1}{c} \dot{\vec{E}}_s = \frac{\vec{\nabla} \times \ddot{\vec{q}} \times r}{c^2 r^2} \qquad \vec{\nabla}\, \ddot{q}\left(\vec{r}\,', t - \frac{|\vec{r} - \vec{r}\,'|}{c}\right)$$

$$= \dddot{q}\left(-\frac{\vec{r} - \vec{r}\,'}{c|\vec{r} - \vec{r}\,'|}\right)$$

— r' schon früher $= \varnothing$

andere Terme sind egal wegen höherer Potenz

$$\to \quad \frac{1}{c} \dot{\vec{E}}_s = -\frac{\vec{r}}{cr} \times \left(\frac{\dddot{\vec{q}} \times \vec{r}}{c^2 r^2}\right)$$

$$\to \quad \vec{E}_s = \frac{(\ddot{\vec{q}} \times \vec{r}) \times \vec{r}}{c^2 r^3} \quad \checkmark$$

$$\vec{E}_s = \vec{B}_s \times \frac{\vec{r}}{r}$$

<u>Abstrahlungsleistung</u>

$$\vec{E}_s = \vec{B}_s \times \frac{\vec{r}}{r}$$

$$\vec{S} = \frac{c}{4\pi} \vec{E}_s \times \vec{B}_s$$

$$-\vec{B}_s \times \left(\vec{B}_s \times \frac{\vec{r}}{r}\right) = -\varepsilon_{ijk} B_j \varepsilon_{klm} B_l \frac{r_m}{r}$$

$$-(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) B_j B_l \frac{r_m}{r} = \underbrace{-B_m B_i \frac{r_m}{r}}_{\substack{\varnothing \\ \text{Transversal}}} + B_l B_l \frac{r_i}{r}$$

$$\to \quad \vec{S} = \frac{c}{4\pi} \vec{B}_s^{\,2} \frac{\vec{r}}{r} \qquad \vec{B}_s = \frac{\ddot{\vec{q}} \times \vec{r}}{c^2 r^2}$$

$$\to \quad \vec{S} = \frac{1}{4\pi} \left(\ddot{\vec{q}} \times \frac{\vec{r}}{r}\right)^2 \frac{\vec{r}}{c^3 r^3} + O\left(\frac{1}{r^3}\right) \checkmark$$

egal weil ∂V mit r^2 geht

$r \to \infty \Rightarrow \frac{1}{r} \to \varnothing$

$$d^2\vec{f} = \frac{\vec{r}}{r} r^2 \, d\Omega$$

$$\oint_{\partial V} d^2\vec{f}\ \vec{S} = -\frac{d}{dt}\left(\vec{A}_{m} + \vec{A}_{f}\right)$$

$$dP = \vec{S}\, d^2\vec{f} = \frac{1}{4\pi c^3}\left(\ddot{\vec{q}} \times \frac{\vec{r}}{r}\right)^2 d\Omega$$

$$\rightarrow \quad \frac{dP}{d\Omega} = \frac{1}{4\pi c^3}\left(\ddot{\vec{q}}^{\,2}\ \underbrace{\cancel{\frac{\vec{r}\vec{r}}{r^2}}}_{1} - \left(\ddot{\vec{q}}\cdot\frac{\vec{r}}{r}\right)^2\right)$$

$$P = \int d\Omega\ \frac{dP}{d\Omega} = \int d\Omega$$

$$q\left(\vec{r}\,', t - \frac{r}{c} + \frac{\vec{r}\vec{r}\,'}{cr}\right) \approx q\left(t - \frac{r}{c}, \frac{\vec{r}}{r}\right)$$

$\rightarrow$ nach $\int d\Omega$ ist $P\left(t - \frac{r}{c}\right)$

Energie, die z. ZP t eine Kugel d Rad r durchsetzt i.d. Zeiteinh.

<u>Berechnung von $\vec{q}$</u>

$$\dot{\vec{q}} = \int d^3r'\ j\Big(\vec{r}\,', \underbrace{t - \frac{r}{c}}_{t_r} + \underbrace{\frac{\vec{r}\vec{r}\,'}{cr}}_{a}\Big)$$

$$\vec{j}\left(\vec{r}\,', t_r + \frac{r\vec{r}\,'}{cr}\right)\Big|_{a=0} = \vec{j}(\vec{r}\,', t_r) + \dot{\vec{j}}\ \frac{\vec{r}\cdot\vec{r}\,'}{cr} + \ldots$$

$$\dot{\rho} + \partial_i' j_i = 0$$

$$\underbrace{\partial_i'(j_i r_k')}_{\text{verschw.}} = r_k' \underbrace{\partial_i' j_i}_{-\dot{\rho}} + j_k$$

$$\underbrace{\partial_i'(\dot{j}_i r_k r_k') r_l'}_{\text{verschw.}} = \partial_i'(\dot{j}_i r_k') r_k r_l' + \dot{j}_i r_k' \partial_i'(r_k r_l')$$

$$= -r_k' \ddot{\rho}\, r_k r_l' + \dot{j}_k r_k r_l' + \dot{j}_l r_k' r_k$$

$$\vec{r}\,' \times \dot{\vec{j}} \times \vec{r} = (\vec{r}\cdot\vec{r}\,')\dot{\vec{j}} - (\dot{\vec{j}}\,\vec{r})\vec{r}\,'$$

$$\rightarrow \quad 2(\vec{r}\cdot\vec{r}\,')\dot{\vec{j}} = r' \times \dot{\vec{j}} \times \vec{r} + \cancel{(\dot{\vec{j}}\,\vec{r})\vec{r}\,'} + \ddot{\rho}\,(\vec{r}\cdot\vec{r}\,')\,\vec{r}\,' - \cancel{(\dot{\vec{j}}\cdot\vec{r})\vec{r}\,'}$$

✓

$$\dot{\vec{q}} = \int d^3r' \left(\cdot \dot{\varrho}\, \vec{r}\,' + \frac{1}{2cr} \left(\vec{r}\,' \times \dot{\vec{j}} \times \vec{r} + \dot{\varrho}\, (\vec{r}\,\vec{r}\,')\, \vec{r}\,' \right) \right)$$

$$\vec{p} = \int d^3r'\, \varrho\, \vec{r}\,'$$

$$\vec{m} = \frac{1}{2c} \int d^3r'\, \vec{r}\,' \times \vec{j}$$

$$\overleftrightarrow{Q} = \int d^3r'\; 3\, x_i' x_j'\, \varrho \qquad \vec{Q} = \overleftrightarrow{Q} \cdot \frac{\vec{r}}{r}$$

$$\longrightarrow \quad \ddot{\vec{q}} = \ddot{\vec{p}} + \ddot{\vec{m}} \times \frac{\vec{r}}{r} + \frac{1}{6c} \dddot{\vec{Q}} \qquad ✓$$

→ Einsetzen in $\vec{E}_S$ & $\vec{B}_S$

Näherung: $\frac{\vec{r}\,\vec{r}\,'}{cr}$ klein $<< T$ / char. Änderungsz. d. Stromd.

$\approx \frac{d}{c}$ / Abmessung. d Quelle

Ladung mit v bew.: $\quad v \approx \frac{d}{T}$

$\longrightarrow \quad v << c$

$c = \lambda f = \lambda \frac{1}{T} \quad \longrightarrow \quad \frac{\lambda}{c} \approx T$

$\longrightarrow \quad d << \lambda \quad$ ✓

Gleichm. Abbremsung eines Teilch.

$$\dot{\vec{R}}(t) = \vec{v}(t) = \begin{cases} v_0\, \vec{e} & t < \emptyset \\ (v_0 - a t)\, \vec{e} & 0 < t < T \\ \emptyset & t > T \end{cases} \qquad T = \frac{v_0}{a}$$

$$\vec{j}(\vec{r}\,', t') = q\, v(t')\, \delta(\vec{r} - \vec{R}(t'))$$

Nur Dipol n. wenn $v_0 << c \quad \rightarrow \quad \ddot{q} \approx \ddot{\vec{p}}$

$$\vec{p} = q\vec{R} \implies \ddot{\vec{q}} = -q\,a\,\vec{e} \qquad 0 < t < T$$

$$\rightarrow \quad P = \frac{1}{4\pi c^3} \int d\Omega \left[\ddot{q}^2 - \left(\frac{\vec{r}}{r} \ddot{q} \right)^2 \right]$$

$\vec{z}$ - Achse in R. $\ddot{\vec{q}}$ $\quad \begin{pmatrix} \cdots \\ \cdots \\ \cos\vartheta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} q\,a$

$$= \frac{1}{4\pi c^3} \int d\Omega \; q^2 a^2 (1 - \cos^2\vartheta)$$

$$\int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \; \sin\vartheta \; (1 - \cos^2\vartheta)$$

$$\rightarrow 2\pi \int_{-1}^{1} d(\cos\vartheta)\,(1 - \cos^2\vartheta) = 2\pi \left(x - \frac{x^3}{3} \right)\Big|_{-1}^{1}$$

$$= 2\pi \left(2 - \frac{2}{3} \right) = \frac{8\pi}{3}$$

$$\rightarrow \quad P = \frac{1}{4\pi c^3} q^2 a^2 \frac{8\pi}{3} = \frac{q^2 a^2}{c^3} \frac{2}{3}$$

Abgestr. Energie $= PT = \dfrac{2}{3} \dfrac{q^2 a^2}{c^3} T \quad \leftarrow \quad T = \dfrac{v_0}{a}$

$$= \frac{2}{3} \frac{q^2}{c^3} \frac{v_0^2}{T}$$

$W_{kin} = \dfrac{m v_0^2}{2}$ (vor Abbr.)

$$\frac{W_r}{W_{kin}} = \frac{2}{3} \frac{q^2}{c^3} \frac{\cancel{v_0^2}}{T} \frac{2}{m \cancel{v_0^2}} = \underbrace{\left(\frac{2}{3} \frac{q^2}{c^3 m} \right)}_{\tau_s} \frac{2}{T}$$

sehr geringe τ_s in Vgl. zu T

$\rightarrow$ wenig Energie abgestrahlt

Abstrahlg bei period. Zeitabh.

$$\rho(\vec r, t) = \frac{1}{2}\rho(\vec r)\, e^{-i\omega t} + c.c.$$

$$\vec j(\vec r, t) = \frac{1}{2}\, j(\vec r)\, e^{-i\omega t} + c.c.$$

$$\vec k = \frac{\omega}{c}\frac{\vec r}{r} \qquad \vec j\Big(\vec r\,', \overbrace{t_r + \frac{\vec r\,\vec r\,'}{c r}}^{t}\Big)$$

$$\vec q = \int d^3r'\, \vec j(\vec r\,', t)$$ umschreiben mit j in Term mit t_r vorne und Rest in $\int$

$$= e^{-i\omega t_r}\frac{1}{2}\int\Big(j(\vec r\,')\, e^{-i\omega\frac{r\vec r\,'}{cr}} + c.c.\Big)$$

$$\vec k\,\vec r\,' = \frac{\omega}{c}\frac{\vec r}{r}\vec r\,'$$

$$\rightarrow \dot{\vec q} = e^{-i\omega(t - \frac{r}{c})}\frac{1}{2}\int j(\vec r\,')\, e^{-i\vec k\vec r\,'} + c.c. \quad \checkmark$$

Taylor d. exp Fkt wenn $\vec k\vec r\,'$ klein

$$|\vec k\vec r\,'| = \frac{\omega}{c}\Big|\frac{\vec r\,\vec r\,'}{r}\Big| \rightarrow 2\pi\frac{d}{\lambda} << 1$$

$$\underbrace{\vec e_r\cdot\vec r\,'}_{d}$$

$$c = \lambda f = \lambda\frac{\omega}{2\pi}$$

$$\rightarrow \frac{\omega}{c} = \frac{2\pi}{\lambda} \qquad e^x = 1 + x + \frac{1}{2}x^2 + \dots$$

$$e^{-i\vec k\vec r\,'} = 1 - i\vec k\vec r\,' - \dots$$

$$\rightarrow \dot{\vec q} = e^{-i\omega(t-\frac{r}{c})}\frac{1}{2}\int j(\vec r\,')\big(1 - i\vec k\vec r\,'\big)\, d^3r'$$

$$= e^{-i\omega t_r}\frac{1}{2}\int j(\vec r\,')\, d^3r' \qquad \text{1. Term}$$

$$- e^{-i\omega t_r}\frac{1}{2}\int i\vec k\vec r\,'\, j(\vec r\,')\, d^3r' \qquad \text{2. Term}$$

1.) $j(\vec r') = \dot\rho\, \vec r'$

$$\rightarrow \quad e^{-i\omega t_r} \frac{1}{2} \int \dot\rho(\vec r')\, \vec r'\, d^3 r'$$

2) $$\rightarrow \quad e^{-i\omega t_r} \frac{1}{2} \int i \frac{\omega \vec r \cdot \vec r'}{c\, r}\, j(\vec r')\, d^3 r'$$

$k = \frac{\omega}{c}$

$$\frac{1}{2}\, \vec r' \times \vec j \times r \quad + \quad \frac{1}{2}\, \dot\rho\, (\vec r \cdot \vec r')\, \vec r'$$

$\vec m'$ $\quad\quad\quad$ $\vec Q$

$\rightarrow \frac{1}{2c}$ $\quad$ Zeitableitung fehlt im 2. Term & bei $\vec m$ fehlt

$i\omega$??

Elektrische Dipolstr.

hier ρ zeitunabh.! Periodische Quellterme

$$\vec p = \int d^3 r'\, \rho(\vec r')\, \vec r'$$

$$\vec p(t_r) = e^{-i\omega t_r} \frac{1}{2} \vec p + c.c.$$

$$\vec q(t_r) = \vec p(t_r) \quad // \text{ Def. Dipolstr.}$$

$$\vec B_s = \frac{\ddot{\vec q} \times r}{c^2 r^2} \qquad \ddot{\vec q} = -\omega^2\, \vec p(t_r)$$

$$= \frac{-\omega^2\, \vec p(t_r) \times r}{c^2 r^2} = \frac{-\omega^2\, e^{-i\omega t_r}}{c^2 r^2} \frac{1}{2}\, \vec p \times \vec r + c.c.$$

$$= -\frac{\omega^2\, e^{-i\omega(t - \frac{r}{c})}}{c^2 r^2} \frac{1}{2}\, \vec p \times \vec r + c.c.$$

$$= \left(\frac{\vec r}{r} \times \vec p\right) \frac{k^2\, e^{-i\omega t}\, e^{ikr}}{r} \frac{1}{2} + c.c.$$

$$\vec E_s = \vec B_s \times \frac{\vec r}{r}$$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c^3}\left[\ddot{\vec{q}} \times \frac{\vec{r}}{r}\right]^2$$

$$= \frac{1}{4\pi c^3}\left[-\omega^2 \vec{p}(t_r) \times \frac{\vec{r}}{r}\right]^2$$

$$= \frac{c^4 k^4}{4\pi c^3}\frac{1}{4}\left[\left(\vec{p}\, e^{-i\omega t_r} + c.c.\right) \times \frac{\vec{r}}{r}\right]^2$$

$$= \frac{c^4 k^4}{4\pi c^3}\frac{1}{4}\Big[\underbrace{\vec{p}\, e^{-i\omega t_r} \times \frac{\vec{r}}{r}}_{a} + \underbrace{\vec{p}^{\,*} e^{i\omega t_r} \times \frac{\vec{r}}{r}}_{b}\Big]^2$$

$$= \frac{c^4 k^4}{4\pi c^3}\frac{1}{4}\left[a^2 + 2ab + b^2\right] = \frac{c k^4}{4\pi}\frac{1}{4}\Big[e^{-2i\omega t_r}\left(\vec{p} \times \frac{\vec{r}}{r}\right)^2$$

$$+ \left|\vec{p} \times \frac{\vec{r}}{r}\right|^2 + c.c.\Big]$$

Zeitmittel $\left\langle \frac{dP}{d\Omega} \right\rangle$ über $T = \frac{2\pi}{\omega}$

$$= \frac{1}{T}\int_0^T \frac{c}{4\pi} k^4 \frac{1}{4}\left\{\left|\vec{p} \times \frac{\vec{r}}{r}\right|^2 + e^{-2i\omega t_r}\left(\vec{p} \times \frac{\vec{r}}{r}\right)^2 + c.c.\right\} dt$$

$$= \frac{1}{T}\frac{c}{4\pi} k^4 \frac{1}{4}\Big\{\underbrace{\left|\vec{p} \times \frac{\vec{r}}{r}\right|^2 t\,\Big|_0^T}_{\left|\vec{p} \times \frac{\vec{r}}{r}\right|^2 \frac{2\pi}{\omega}} + \underbrace{\frac{e^{-2i\omega t_r}}{-2i\omega}\left(\vec{p} \times \frac{\vec{r}}{r}\right)^2\Big|_0^T}_{\underbrace{e^{-2i\cancel{\omega}\frac{2\pi}{\cancel{\omega}}}}_{e^{-4\pi i}\,=\,1} \cdot C - e^0 \cdot C} + c.c.\Big\}$$

$\longrightarrow$ Term wird 0

$$\rightarrow \left\langle \frac{dP}{d\Omega} \right\rangle = \frac{\cancel{\omega}}{\cancel{2\pi}}\frac{c}{4\pi} k^4 \frac{1}{4}\frac{\cancel{2\pi}}{\cancel{\omega}}\left|\vec{p} \times \frac{\vec{r}}{r}\right|^2 + c.c.$$

$$= \frac{c}{2\pi} k^4 \frac{1}{4}\left|\vec{p} \times \frac{\vec{r}}{r}\right|^2$$

Hat $\vec{p}$ feste Richtung $\vec{p}$ zB $\vec{e}_z$

$$|\vec{p}\times\frac{\vec{r}}{r}|^2 = |p\,\vec{e}_z\times\frac{\vec{r}}{r}|^2 = p^2\left|\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}\sin\vartheta\cos\varphi\\ \sin\vartheta\sin\varphi\\ \cos\vartheta\end{pmatrix}\right|^2$$

$$= p\left|\begin{pmatrix}-\sin\vartheta\sin\varphi\\ \sin\vartheta\cos\varphi\\ 0\end{pmatrix}\right| = p^2(\sin^2\vartheta\sin^2\varphi+\sin^2\vartheta\cos^2\varphi)$$

$$= p^2\sin^2\vartheta$$

$$\rightarrow \left\langle\frac{dP}{d\Omega}\right\rangle = \underbrace{\frac{c}{2\pi}k^4\frac{1}{4}p^2}_{A}\sin^2\vartheta$$

$$\int d\Omega\left\langle\frac{dP}{d\Omega}\right\rangle = A\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\vartheta\, d\vartheta\,\underbrace{\sin^2\vartheta}_{1-\cos^2\vartheta}$$

$$= A\,2\pi\underbrace{\int_{-1}^{1}d(\cos\vartheta)(1-\cos^2\vartheta)}_{\frac{8\pi}{3}}$$

$$= \frac{c}{\cancel{2\pi}}k^4\frac{1}{\cancel{4}}p^2\,\frac{\cancel{8\pi}^{\,4}}{3} = \frac{c\,k^4p^2}{3}\ \checkmark$$

Magn Dipolstrahlung

analog nur mit $\vec{m} = \frac{1}{2c}\int d^3r'\,\vec{r}'\times\vec{j}(\vec{r}')$

$$\vec{q} = \frac{1}{2}e^{-i\omega t_r}\vec{m}\times\frac{\vec{r}}{r} + c.c.$$

$$\vec{B}_s = \frac{\ddot{\vec{q}}\times\vec{r}}{c^2r^2} \qquad \ddot{\vec{q}} = -\omega^2\frac{1}{2}e^{-i\omega t_r}\vec{m}\times\frac{\vec{r}}{r}+c.c.$$

Dann $\frac{dP}{d\Omega} = \frac{1}{4\pi c^3}\left[\ddot{\vec{q}}\times\frac{\vec{r}}{r}\right]^2$

Einsetzen, Zeitmittel, $\vec{m}$ in z Richtg, über $d\Omega$ $\int$ ✓

Quadrupolstr.

$$Q_{ij} = \int d^3r' \, 3 r_i' r_j' \, \rho(\vec r')$$

$$\vec q = - e^{-i\omega t_r} \frac{ik}{6} \frac{\vec r}{r} \frac{1}{2} Q_{ij} + c.c.$$

$$e^{-i\omega t_r}$$

$$\vec B_r = \frac{\ddot{\vec q} \times \vec r}{c^2 r^2}$$

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c^3} \left(\ddot{\vec q} \times \frac{\vec r}{r} \right)^2$$

Zeitmittel $\left\langle \frac{dP}{d\Omega} \right\rangle$ Koordinatenachsen → Hauptachsen

zeitunabh.

$$Q_{ij} \to \begin{pmatrix} Q_{11} & & \\ & Q_{22} & \\ & & Q_{33} \end{pmatrix}$$

$$\ddot{\vec q} = -\omega^2 \vec q$$

$$\to \left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{2\pi} \frac{k^6}{36} \frac{1}{4} \left| \frac{\vec r}{r} \cdot \overleftrightarrow{Q} \times \frac{\vec r}{r} \right|^2$$

$$\begin{pmatrix} \sin\vartheta\cos\varphi \\ \sin\vartheta\sin\varphi \\ \cos\vartheta \end{pmatrix} \qquad // \; x_1 = y_i A_{1i}$$

$$\begin{pmatrix} Q_{11} & & \\ & Q_{22} & \\ & & Q_{33} \end{pmatrix} \begin{pmatrix} Q_{11} \sin\vartheta\cos\varphi \\ Q_{22} \sin\vartheta\sin\varphi \\ Q_{33} \cos\vartheta \end{pmatrix} \times \begin{pmatrix} \sin\vartheta\cos\varphi \\ \sin\vartheta\sin\varphi \\ \cos\vartheta \end{pmatrix}$$

$$= \begin{pmatrix} Q_{22} \sin\vartheta\sin\varphi \cdot \cos\vartheta - Q_{33}\cos\vartheta \cdot \sin\vartheta\sin\varphi \\ Q_{33}\cos\vartheta \sin\vartheta\cos\varphi - Q_{11}\sin\vartheta\cos\varphi\cos\vartheta \\ Q_{11}\sin\vartheta\cos\varphi\sin\vartheta\sin\varphi - Q_{22}\sin\vartheta\sin\varphi\sin\vartheta\cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} (Q_{22} - Q_{33}) \sin\vartheta\sin\varphi\cos\vartheta \\ (Q_{33} - Q_{11}) \quad -"- \\ (Q_{11} - Q_{22}) \sin^2\vartheta \sin\varphi\cos\varphi \end{pmatrix}$$

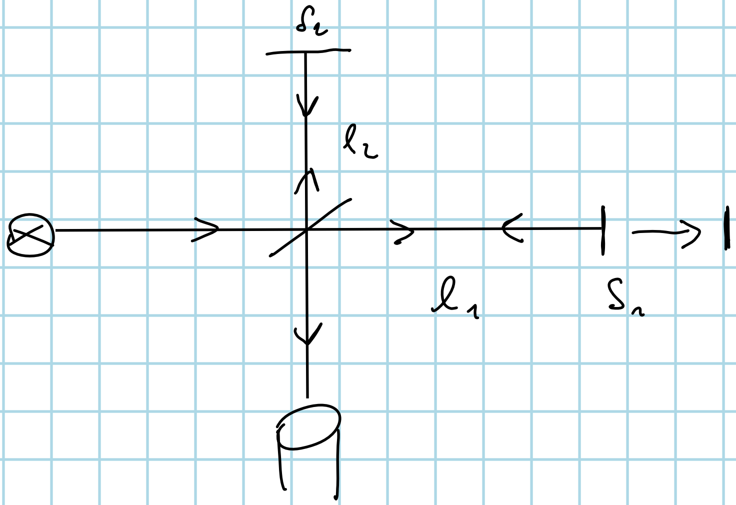
$$\rightarrow \; |Q_{33} - Q_{11}|^2 \sin^2\vartheta \sin^2\varphi \cos^2\vartheta$$

$$+ \; \dots \; + \; ---$$

Juhuu!

Relativitätstheorie

Morley Michelson



Versuchsaufbau ruht im Äther

$t_1 = \frac{2l_1}{c}$ $\quad t_2 = \frac{2l_2}{c}$ $\quad l_1 = l_2$ dann $t_1 = t_2$

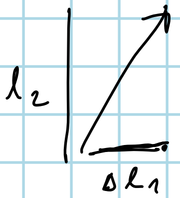
keine Interferenzen

Versuchsaufbau bewegt sich in Richtung S_1

2 Verschiebungen Δl_1-hin, Δl_2-rück $\quad v$ Geschw. v Aufbau

$$t_{1h} = \frac{l_1 + \Delta l_{1h}}{c} = \frac{\Delta l_{1h}}{v} \qquad \Big| \qquad t_{1r} = \frac{l_1 - \Delta l_{1r}}{c} = \frac{\Delta l_{1r}}{v}$$

$$\frac{l_1 + \Delta l_{1h}}{c} = \frac{\Delta l_{1h}}{v}$$



$$l_1 v + \Delta l_{1h} v = \Delta l_{1h} c$$

$$\frac{l_1 v}{(c-v)} = \Delta l_{1h} \qquad \Big| \qquad \Delta l_{1r} = \frac{l_1 v}{c+v}$$

$$t_{1h} = \frac{l_1 \cancel{v}}{(c-v)\cancel{v}} = \frac{l_1}{c-v} \qquad t_{1r} = \frac{l_1}{c+v}$$

$$t_{1h} + t_{1r} = \frac{l_1(c+v) + l_1(c-v)}{(c-v)(c+v)} = \frac{2 l_1 c}{c^2 - v^2}$$

$$= \frac{2 l_1}{c\left(1 - \frac{v^2}{c^2}\right)} = t_{ges\,1}$$

$$t_{2h} = \frac{\sqrt{l_2^2 + \Delta l_{1h}^2}}{c} = \frac{\Delta l_{1h}}{v} \qquad t_{2r} = t_{2h}$$

$$\frac{l_2^2 + \Delta l_{1h}^2}{c^2} = \frac{\Delta l_{1h}^2}{v^2} \qquad l_2^2 v^2 + \Delta l_{1h}^2 v^2 = \Delta l_{1h}^2 c^2$$

$$\Delta l_{1h}^2 = \frac{l_2^2 v^2}{c^2 - v^2}$$

$$\Delta l_{1h} = \frac{l_2 v}{\sqrt{c^2 - v^2}}$$

$$t_{2h} = \frac{l_2}{\sqrt{c^2 - v^2}} \qquad t_{2ges} = 2\frac{l_2}{\sqrt{c^2 - v^2}} = \frac{2 l_2}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$l_1 = l_2 = L \rightarrow t_{1ges} > t_{2ges}$$

$\rightarrow$ Interferenz

Gibt es aber nicht $\rightarrow$ kein Äther

→ Erklärungsversuch L-Kontraktion um $\sqrt{1-\beta^2}$

Zeit-Dill.

$$t_{2a} = \frac{2l_2}{c} \qquad t_{2b} = \frac{2l_2}{c\sqrt{1-\beta^2}}$$

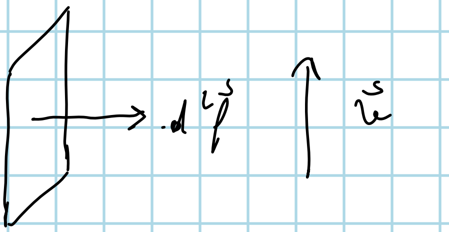
→ Damit das gleiche rauskommt mit $\sqrt{1-\beta^2}$ multiplizieren!

Lorentztrafo

1) c überall gleich; 2) alle Bezugssysteme äquivalent; 3) Raum homogen & isotrop

1) → lineare Gl.

2)



S_{ruht} $S_{bew.}$ Ebene ist gleich

3) c gleich

$c^2t^2 = x^2 + y^2 + z^2$

& $c^2t'^2 = x'^2 + y'^2 + z'^2$

wegen konst. c

(gleicher Ursprung)

$$s^2 = c^2t^2 - x^2 - y^2 - z^2$$

bei Trafo: wenn $s^2 = 0 \Rightarrow s'^2 = 0$

dann $s^2 = A s'^2$

Skalentrafo kann nur von $\vec{v}$ abh.

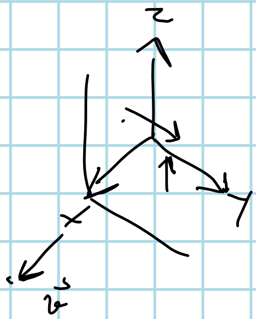
Isotropie: nur $|\vec{v}|$ → für Umkehrtrafo dasselbe!

→ $AA^{-1} = 1$ $A^{-1} = A$ → $A^2 = 1$ $A(|\vec{v}| \to 0) = 1$

& $A = 1$

Trafo: S' entlg x-Achse zu S

$\vec{e}_x = \vec{e}_{x'}$



$y = 0$ & $z = 0$ in $y' = 0$ & $z' = 0$

$\rightarrow \quad y = y' \quad z = z'$

$x' = A(x - vt)$

$t' = Bt - Dx$

$c^2 t'^2 - x'^2 = c^2 t^2 - x^2$

$c^2 (Bt - Dx)^2 - (A(x - vt))^2 = c^2 t^2 - x^2$

$c^2 (B^2 t^2 - 2BtDx + D^2 x^2) - A^2 (x^2 - 2xvt + v^2 t^2) = c^2 t^2 - x^2$

1) x^2: $\quad c^2 D^2 - A^2 = -1$

2) t^2: $\quad c^2 B^2 - A^2 v^2 = c^2 \quad | : c^2$

3) tx: $\quad -c^2 \cancel{2} BD + \cancel{2} A^2 v = 0$

3) $\quad \dfrac{A^2 v}{B c^2} = D \quad \rightarrow 1$

1) $\quad c^2 \dfrac{A^4 v^2}{B^2 c^4} - A^2 = -1$

$= A^2 \left(\dfrac{A^2 v^2}{B^2 c^2} - 1 \right) = -1$

$= \dfrac{A^2}{B^2} \Big(\underbrace{B^2 - A^2 \frac{v^2}{c^2}}_{1} \Big) = 1 \quad \rightarrow \quad A = B$

$\rightarrow \quad A^2 - A^2 \dfrac{v^2}{c^2} = 1$

$A = \dfrac{1}{\underbrace{\sqrt{1 - \frac{v^2}{c^2}}}_{\gamma}} = B \qquad \beta = \dfrac{v}{c}$

$\dfrac{A^{\cancel{2}} v}{\cancel{A} c^2} = D \quad \rightarrow \quad D = \dfrac{v}{c^2} \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Trafo
$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\Lambda} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$(\Lambda(v))^{-1} = \Lambda(-v)$$

$$\rightarrow \begin{pmatrix} \gamma & \beta\gamma & - & - \\ \beta\gamma & \gamma & - & - \\ - & - & - & - \\ - & - & - & - \end{pmatrix} = \Lambda^{-1}$$

Allg. Lorentz Trafos:

Zeitspieg.: $\Lambda = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ Raumspg.: $\Lambda = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$\det \Lambda = -1 \longrightarrow$ keine eig. Lo. Trafos

allg. Lor.-Tr.: 2 Drehungen & 1 stod. Lor.

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & D_{ij}(\alpha_1', \alpha_2', \alpha_3') & \\ 0 & & & \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & \cdot & \cdot \\ -\beta\gamma & \gamma & - & - \\ - & - & 1 & - \\ - & - & \cdot & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & D_{ij}(\alpha_1, \alpha_2, \alpha_3) & \\ 0 & & & \end{pmatrix}$$

Drehung in x-R. — gesch. Trafo x-R. — Drehung in ges. Richtg.

Vierergr.

geom. Int.: Punkte im 4er R. sind Ereignisse

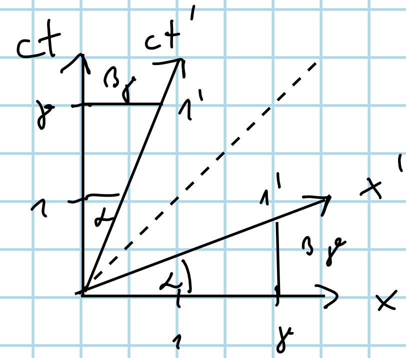
jedes T d. Weltlinie Welt $= \Sigma$ Weltlinien

physik Ges. — geom Bez. zw. Weltlinien

Geom Darst.: nur auf ct & x besch.

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} ct' + \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} x'$$

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \gamma \\ \beta\gamma \end{pmatrix} ct' + \begin{pmatrix} \beta\gamma \\ \gamma \end{pmatrix} x'$$



Steigung / Steilheit hängt von β ab!

Winkel α: $\tan\alpha = \frac{\beta\gamma}{\gamma} = \beta$

$\rightarrow \alpha = \arctan\beta$ ✓

Menge aller x' EV: $\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \beta\gamma \\ \gamma \end{pmatrix}$ ✓ anders wie man Betr. macht!!

$$-c^2t^2 + x^2 = -\beta^2\gamma^2 + \gamma^2$$

$$= \gamma^2(1-\beta^2) = 1 \checkmark$$

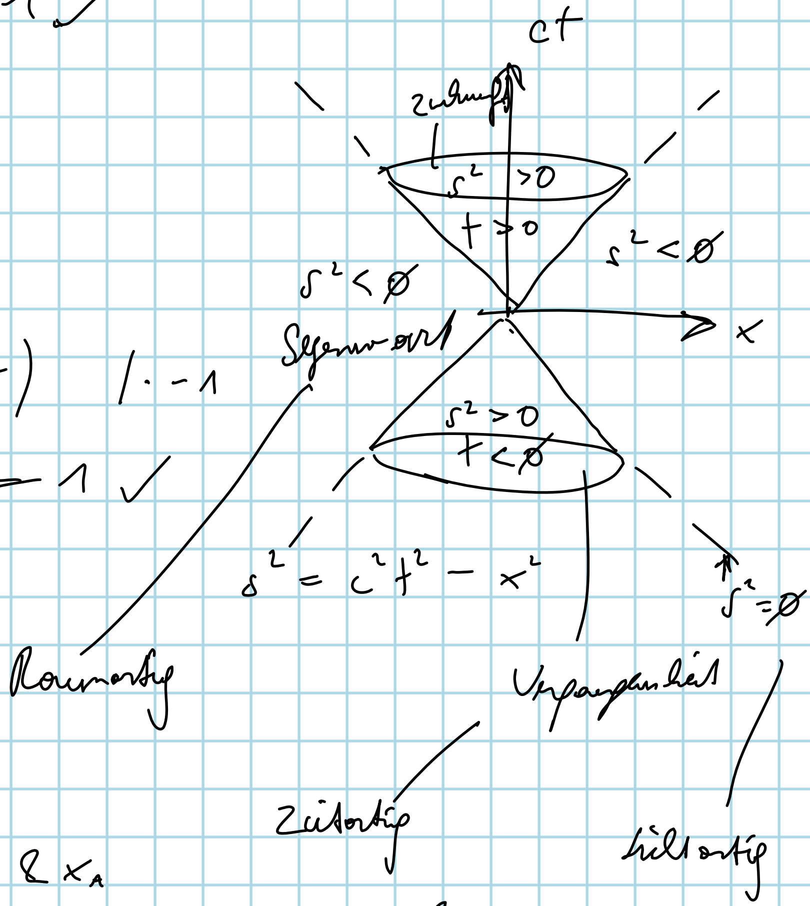
$c^2t^2 = x^2$

ct' EV $\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \gamma \\ \beta\gamma \end{pmatrix}$

$$-c^2t^2 + x^2 = -\gamma^2 + \beta^2\gamma^2$$

$$-c^2t^2 + x^2 = \gamma^2(-1+\beta^2) \quad | \cdot -1$$

$$c^2t^2 - x^2 = \gamma^2(1-\beta^2) = 1 \checkmark$$



Längenkontr., Zeitdilat.

Längenmess.

Maßst. ruht in S $\quad x_E$ & x_A

$$\begin{pmatrix} ct' \\ x'_E \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct_E \\ x_E \end{pmatrix}$$

Messung zur gl. Zeit t' in S'

$$\begin{pmatrix} ct' \\ x'_A \end{pmatrix} = \begin{pmatrix} - " - \end{pmatrix} \begin{pmatrix} ct_A \\ x_A \end{pmatrix}$$

Zeiten wo d. Maßstab ruht sind beliebig

$\rightarrow$ gleichs. $\quad \gamma ct_E - \beta\gamma x_E = \gamma ct_A - \beta\gamma x_A$

$$\rightarrow \quad (x_E - x_A)\,\beta\gamma = \gamma c(t_E - t_A')$$

$$x_E' = -\beta\gamma c t_E + \gamma x_E$$

$$x_A' = -\beta\gamma c t_A + \gamma x_A$$

$$(x_E' - x_A') = -\beta\gamma c(t_E - t_A) + \gamma(x_E - x_A)$$

$$(\;-"-) - \gamma(x_E - x_A) = \beta\gamma c(t_E - t_A) \quad / \cdot \beta$$

$$(-"-)\frac{1}{\beta} - \frac{\gamma}{\beta}(-"-) = \gamma c(-"-)$$

$$\rightarrow (x_E - x_A)\,\beta\gamma = (x_E' - x_A')\frac{1}{\beta} - \frac{\gamma}{\beta}(x_E - x_A)$$

$$(x_E - x_A)\left[\beta\gamma + \frac{\gamma}{\beta}\right] = (x_E' - x_A')\frac{1}{\beta} \quad / \cdot \beta$$

$$(x_E' - x_A') = (x_E - x_A)\left[\gamma + \beta^2\gamma\right]$$

$$= \underbrace{\gamma\underbrace{\left[1 + \beta^2\right]}_{\gamma^{-2}}}_{\gamma^{-1}}(x_E - x_A) \quad \checkmark$$

<u>Zeitmessung</u> am gleichen Ort x' — Orte d. Zeitmess. x_A & x_E u. t_A & t_E beliebig

$$\begin{pmatrix} ct_E' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} ct_E \\ x_E \end{pmatrix}$$

$$\begin{pmatrix} ct_A' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} ct_A \\ x_A \end{pmatrix}$$

$$ct_E' = \gamma c t_E - \beta\gamma x_E \qquad c(t_E' - t_A') = \gamma c(t_E - t_A) - \beta\gamma(x_E - x_A)$$

$$ct_A' = \gamma c t_A - \beta\gamma x_A$$

$$x' = -\beta\gamma c t_E + \gamma x_E \qquad \gamma(x_E - x_A) = \beta\gamma c(t_E - t_A)$$

$$x' = -\beta\gamma c t_A + \gamma x_A$$

$$\rightarrow \cancel{c}(t_E' - t_A') = \gamma\cancel{c}(t_E - t_A) - \beta^2\gamma\cancel{c}(t_E - t_A)$$

$$\Rightarrow (t_E' - t_A') = \underbrace{\gamma \underbrace{(1-\beta^2)}_{\gamma^{-2}}}_{\gamma^{-1}} (t_E - t_A)$$ ✓

Vierervektoren $\quad \vec{a} = \sum_\nu a^\nu \vec{e}_\nu$ geht v 0 – 3

Ortsvektor analog $\quad \vec{x} = x^\mu \vec{e}_\mu = ct\, \vec{e}_t + x\, \vec{e}_x + y\, \vec{e}_y + \dots$

$$x^\mu = (ct, x, y, z)$$

Def d inneren Prod : Metriktensor

$$\vec{a}\,\vec{b} = a^\mu \vec{e}_\mu\, b^\eta \vec{e}_\eta = a^\mu b^\eta \underbrace{(\vec{e}_\mu, \vec{e}_\eta)}_{g_{\mu\eta}}$$

$$s^2 = c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2$$

Vierer abstand bleibt gleich $\rightarrow$ Metrik :

$$g_{\mu\eta} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad \vec{x}\,\vec{x} = x^\mu x^\eta g_{\mu\eta} = s^2$$

Duale Basis : $\quad \vec{e}_\mu \vec{e}^\eta = \delta_\mu^\eta \quad$ Orthogon.

$$\rightarrow \vec{a} = \underset{\text{kontravar. Komp}}{a^\mu} \vec{e}_\mu = \underset{\text{kovar. Komp}}{a_\mu} \vec{e}^\mu$$ — duale Basis

Inneres Prod. : Metrik $\qquad$ Skalarprod: Vektor · lin. abbild.

(kontra · kovar)

Unterschd. nachschauen

durch EV darstellen:

$$\vec{\vec{e}}^{\eta} = g^{\eta\sigma} \vec{\vec{e}}_{\sigma} \qquad \vec{e}_{\mu} \vec{\vec{e}}^{\eta} = \vec{\vec{e}}_{\mu} g^{\eta\sigma} \vec{\vec{e}}_{\sigma} = g_{\mu\sigma} g^{\eta\sigma} = \delta_{\mu}^{\eta} \checkmark$$

$$\rightarrow g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad g^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu}$$

Konj.: $a^{\mu} = g^{\mu\nu} a_{\nu} \qquad a_{\mu} = g_{\mu\nu} a^{\nu}$

Inneres Prod. $\vec{x}\,\vec{\vec{y}} = x^{\mu} y^{\eta} g_{\mu\eta} = x^{\mu} y_{\eta} \checkmark$

Verh. bei Lorentztrafo

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} \qquad \Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \checkmark$$

$$a'_{\mu} = \Lambda_{\mu}{}^{\nu} a_{\nu}$$

$$t'^{\mu\nu} = \Lambda^{\mu}{}_{\sigma} \Lambda^{\nu}{}_{\xi} t^{\sigma\xi} \checkmark \qquad \Lambda_{\mu}{}^{\nu} = g_{\mu\sigma} \Lambda^{\sigma}{}_{\xi} g^{\nu\xi}$$

$$t'_{\mu\nu} = \Lambda_{\mu}{}^{\sigma} \Lambda_{\nu}{}^{\xi} t_{\sigma\xi} \checkmark$$

Vierergeschw. / Viererbeschl.

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2$$

$$ds = c\, d\tau \qquad ds^2 = c^2 d\tau^2$$

$$d\tau^2 = \frac{ds^2}{c^2} \qquad \tau_{AB} = \frac{1}{c}\int_A^B ds = \tau'_{AB}$$

$$d\tau^2 = \frac{ds^2}{c^2} = dt^2 - \frac{dx^2}{c^2} - \frac{dy^2}{c^2} - \frac{dz^2}{c^2}$$

$$= dt^2 \left(1 - \frac{1}{c^2}\left(\frac{dx^2 + dy^2 + dz^2}{dt^2}\right)\right)$$

$$= dt^2 \left(1 - \frac{u^2}{c^2}\right) = dt^2 \gamma^{-2}$$

$$d\tau = \frac{dt}{\gamma} \qquad \vec{u} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$

<u>Vierergeschw.:</u>

$$u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt}\gamma$$

Zeitartig!!

$$= \begin{pmatrix} c \\ \vec{u} \end{pmatrix} \gamma$$

$$u^\mu u_\mu = \gamma^2 (c^2 - \vec{u}^2) = \gamma^2 c^2 \left(1 - \frac{u^2}{c^2}\right) = c^2 \checkmark$$

<u>Viererbeschl.:</u>

$$a^\mu = \frac{du^\mu}{d\tau} = \frac{d^2 x^\mu}{d\tau^2}$$

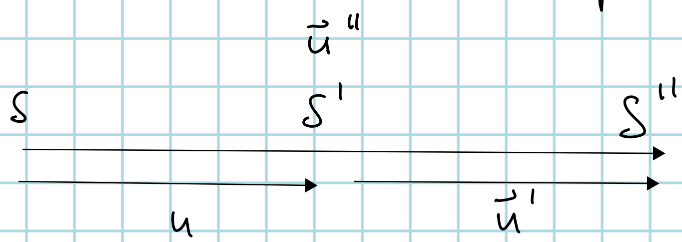
$$u^\mu u_\mu = c^2$$

$$a^\mu u_\mu = \emptyset \qquad \text{// beide Seiten ableiten}$$

$$\rightarrow \quad \vec{\vec{a}} \perp \vec{\vec{u}} \qquad \text{raumartig!}$$

<u>relativist. Add von Geschw.</u>

Berechn mittl. Lorentz-Trafo



in x-Richtg

$$x''^\mu = \Lambda^\mu{}_\eta(u')\, x'^\eta = \Lambda^\mu{}_\eta(u')\, \Lambda^\eta{}_\tau(u)\, x^\tau = \Lambda^\mu{}_\tau(u'')\, x^\tau$$

$$\rightarrow \quad \Lambda^\mu{}_\eta(u')\, \Lambda^\eta{}_\tau(u) = \Lambda^\mu{}_\tau(u'')$$

$$\begin{pmatrix} \gamma' & -\beta'\gamma' & & \\ -\beta'\gamma' & \gamma' & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} \gamma'' & -\beta\gamma'' & & \\ -\beta''\gamma'' & \gamma'' & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\begin{pmatrix} \gamma'\gamma + \beta'\gamma'\beta\gamma & -\gamma'\beta\gamma - \beta'\gamma'\gamma & 0 & 0 \\ -\beta'\gamma'\gamma - \gamma'\beta\gamma & \beta\gamma\beta'\gamma' + \gamma\gamma' & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\gamma'\gamma + \beta'\beta\gamma'\gamma = \gamma'' \longrightarrow \gamma'\gamma(1+\beta'\beta) = \gamma''$$

$$-\beta\gamma'\gamma - \beta'\gamma'\gamma = -\beta''\gamma'' \longrightarrow \gamma'\gamma(\beta+\beta') = \beta''\gamma''$$

$$\longrightarrow \beta'' \cancel{\gamma'\gamma}(1+\beta'\beta) = \cancel{\gamma'\gamma}(\beta+\beta')$$

$$\Rightarrow \beta'' = \frac{\beta+\beta'}{1+\beta'\beta} \checkmark$$

$$\frac{u''}{\cancel{c}} = \frac{\frac{u}{\cancel{c}} + \frac{u'}{\cancel{c}}}{1+\frac{u'u}{c^2}} \qquad u'' = \frac{u+u'}{1+\frac{u'u}{c^2}}$$

$$u'' < u+u'$$

$$u'' = \frac{\frac{3}{4}c + \frac{3}{4}c}{1+\frac{9}{16}} = \frac{\frac{6}{\cancel{4}}}{\frac{25}{\cancel{16}\,4}} = \frac{24}{25}c$$

u'' ist immer $< c$!

<u>Berechnung mittels Weltlinie</u>

2 Systeme S & S' $\xrightarrow[v]{}$ Punktteilch. entl. Weltlinie

u in S u' in S'

Trafo zwischen u & u'

$x^\nu = \Lambda^\nu{}_\eta(-v)\, x'^\eta$ in diff Form

$$\begin{pmatrix} c\,dt \\ dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \gamma & \overset{-\text{ mit } v}{\beta\gamma} & & \\ \beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} c\,dt' \\ dx' \\ dy' \\ dz' \end{pmatrix} = \begin{pmatrix} \gamma c\,dt' + \beta\gamma\,dx' \\ \beta\gamma c\,dt' + \gamma\,dx' \\ dy' \\ dz' \end{pmatrix}$$

$$\cancel{c}\, dt = \gamma \cancel{c}\, dt' + \frac{\beta}{c}\gamma\, dx' \qquad \beta c = \frac{v}{\cancel{c}} \cdot \cancel{c} = v$$

$$dx = \beta\gamma c\, dt' + \gamma\, dx'$$

$$\frac{dx}{dt} = \frac{\beta \cancel{\gamma} c\, dt' + \cancel{\gamma} dx'}{\cancel{\gamma} dt' + \frac{\beta}{c}\cancel{\gamma} dx'} = \frac{\beta c\, dt' + dx'}{dt' + \frac{\beta}{c} dx'} = \frac{\cancel{dt'}\left(\overset{v}{\beta c} + \frac{dx'}{dt'}\right)}{\cancel{dt'}\left(1 + \frac{\beta}{c}\frac{dx'}{dt'}\right)} \quad ✓$$

$$\frac{dy}{dt} = \frac{dy'}{\gamma\, dt' + \frac{\beta}{c}\gamma\, dx'} = \frac{dy'}{\gamma\, dt'\left(1 + \frac{\beta}{c}\frac{dx'}{dt'}\right)} = \frac{u'_y}{\gamma\left(1 + \frac{\beta}{c}u'_x\right)}$$

$\frac{dz}{dt}$ analog

Gleichgerichtete Geschw: analog zu Ber. mit Lorenz

einfacher mittels Lorenz Trafo

$$\begin{pmatrix} \gamma & -\beta\gamma & | & | \\ -\beta\gamma & \gamma & | & | \\ - & - & - & - \end{pmatrix}$$

$$u^\mu = \Lambda^\mu{}_\eta(-v)\, u'^\mu \qquad u^\mu = \gamma(u)\begin{pmatrix} c \\ \vec{u} \end{pmatrix}$$

$$u'^\mu = \gamma(u')\begin{pmatrix} c \\ \vec{u}' \end{pmatrix}$$

// (49) mit Umkehrtrafo

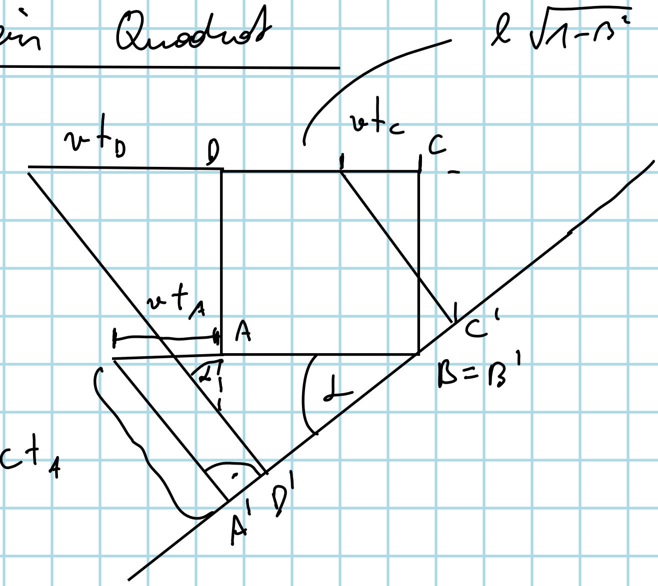
$$1)\ u^0 = \Lambda^0{}_\eta(-v)\, u'^\mu = \gamma(v)\left(u'^0 + \frac{v}{c}\, u'^1\right) \qquad \text{// jetzt f. us einsetzen}$$

$$1)\ u^1 = \gamma(v)\left(+\frac{v}{c}\, u'^0 + u'^1\right)$$

$$\gamma(u)\, c = \gamma(v)\,\gamma(u')\left(c + \frac{v}{c}u_x\right)$$

$$\underline{\gamma(u) = \gamma(v)\,\gamma(u')\left(1 + \frac{v}{c^2}u_x\right)} \quad ✓$$

Superschnappschuss f. ein Quadrat



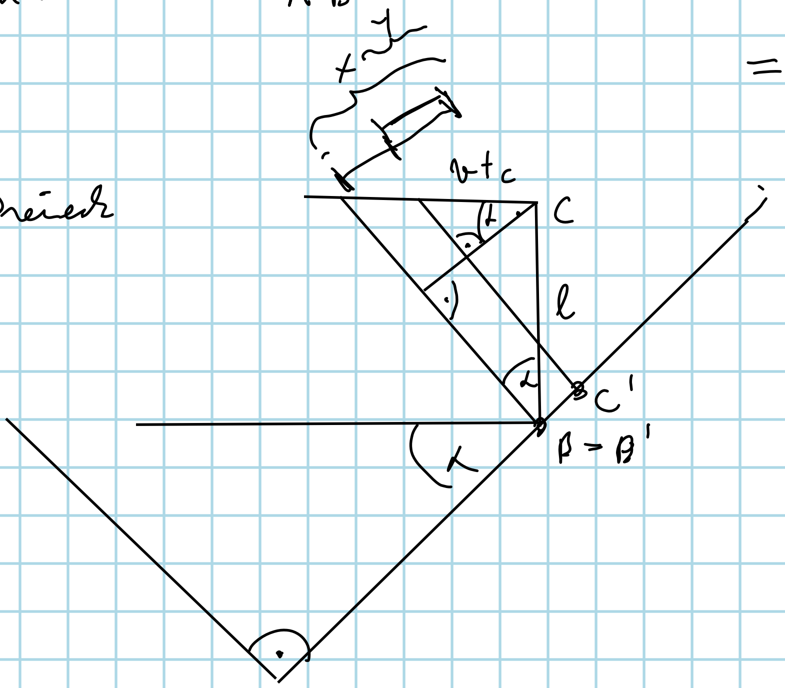
$$ct_A = \left(l\sqrt{1-\beta^2} + vt_A\right)\sin\alpha$$

$$t_A(c - v\sin\alpha) = l\sqrt{}\,\sin\alpha$$

$$ct_A = \frac{l\sqrt{}\,\sin\alpha}{1-\beta\sin\alpha}$$

1) $\tan\alpha = \dfrac{ct_A}{\overline{AB}} \longrightarrow \overline{A'B'} = \dfrac{ct_A}{\tan\alpha} = \dfrac{l\sqrt{}\,\cos\alpha}{1-\beta\sin\alpha}$ ✓

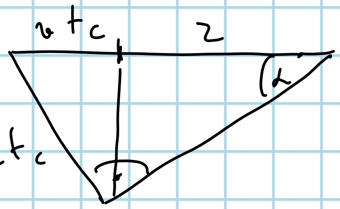
Dreieck

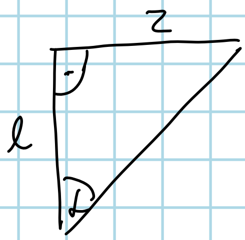


$x = \sin\alpha\; l$ $\qquad x - y = \overline{B'C'}$

$y = \cos\alpha\; vt_c$ $\qquad t_c$ benötigt !!

ct_c:





$\cot\alpha = \dfrac{z}{l}$

$z = l\cot\alpha$

$\frac{\cos}{\sin}$

$$(vt_c + z)\cdot\sin\alpha = ct_c \Longleftarrow$$

$$ct_c = vt_c\sin\alpha + l\cos\alpha \;✓$$

$$t_c(c - v\sin\alpha) = l\cos\alpha$$

$t_c = \dfrac{1}{c}\dfrac{l\cos\alpha}{\left(1-\frac{v}{c}\sin\alpha\right)}$ // oben einsetzen!

$$x - y = l \sin\alpha - v t_c \cos\alpha$$

$$= l \sin\alpha - \frac{v}{c}\,\frac{l\cos^2\alpha}{1 - \frac{v}{c}\sin\alpha} = \frac{c\,l\sin\alpha(1-\beta\sin\alpha) - v\,l\cos^2\alpha}{c(1-\beta\sin\alpha)}$$

$$= \frac{c\,l\sin\alpha - \overbrace{c\,l\,\beta}^{v\,l}\sin^2\alpha - v\,l\cos^2\alpha}{c(1-\beta\sin\alpha)}$$

$$= \frac{c\,l\sin\alpha - v\,l}{c(1-\beta\sin\alpha)} = l\,\frac{(c\sin\alpha - v)}{c(-\text{"}-)}$$

c kürzen → richtig!

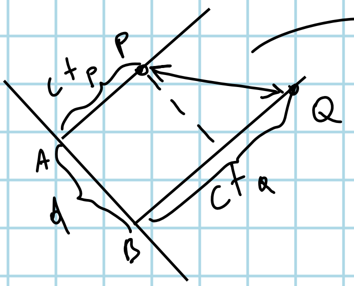
$$\overline{A'B'} = l\,\frac{\sqrt{1-\beta^2}\cos\alpha}{1-\beta\sin\alpha} = l\cos\alpha_0$$

$$\overline{B'C'} = l\,\frac{(\sin\alpha - \beta)}{1-\beta\sin\alpha} = l\sin\alpha_0$$

ausrechnen: $\cos^2\alpha_0 + \sin^2\alpha_0 = 1$ ✓

Satz von Terrell

Alle Sphärenschnappschüsse in einem Raum-Zeitpunkt von Beob. in bel. Inertialsystemen sind gleich!



räuml. Abstand:

$$\sqrt{d^2 + (ct_Q - ct_P)^2}$$

Lorentz-invarianter 4er Abstand f. Sphärenschnappsch. eines bzg. Obj. ruhenden Photographen

$$s^2_{AB} = \underbrace{c^2(t_a - t_b)^2}_{\emptyset} - (\vec{x}_A - \vec{x}_B)^2 = -d^2$$

4er Abstand bel. Pkte auf LS: // raum Abst. eingesetzt

$$s^2_{PQ} = c^2(t_P - t_Q)^2 - d^2 + c^2(t_Q - t_P)^2 = -d^2$$

s ist Lorentz Invariant

→ bei gZ in S' ist $(\text{Diff} = \emptyset)$ muss

$\phi' = \phi$ sein!

gilt f. alle Pkte d Weltlinie

→ Photonenleiter steht normal auf räuml Richtg $\vec{PQ}$

→ auch in S' Sugerschngsschuss

VIII Relativ Mechanik

Punktteilchen

$$p^\mu = m u^\mu = m\,\gamma(u) \begin{pmatrix} c \\ \vec{u} \end{pmatrix}$$

$$\vec{p} = \underbrace{m\,\gamma(u)}_{m(u)}\, \vec{u} \qquad m(\emptyset) = \text{Ruhemasse}$$

Betrag v $p^\mu = m^2 c^2$

$$p^\mu p_\mu = m^2 c^2 = p^0 p_0 - \vec{p}^{\,2}$$

$$\rightarrow p^0 p_0 = m^2 c^2 + \vec{p}^{\,2}$$

$$p^0 = \sqrt{m^2 c^2 + \vec{p}^{\,2}}$$

Rel. BWGL

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \text{relativ:} \quad F^\mu = \frac{d}{d\tau}\, p^\mu$$

Zus. zw räuml. K. v F^μ & $\vec{F}$

$$F^i = \frac{dp^i}{d\tau} = \underbrace{\frac{dt}{d\tau}}_{\gamma!!} \frac{dp^i}{dt} = \gamma (\vec{F})^i$$

$\gamma = \sqrt{1 - \vec{v}}$

Punktteilchen

$$\frac{d}{dt}\left(1-\left(\frac{\dot{\vec{x}}}{c}\right)^2\right)^{-\frac{1}{2}} = -\frac{1}{2}\left(\ldots\right)^{-\frac{3}{2}}\left(-2\left(\frac{\dot{\vec{x}}}{c}\right)\right)\cdot\frac{\ddot{\vec{x}}}{c}$$

$$= \frac{\vec{v}\,\vec{a}\,\gamma^3}{c^2}$$

$$F^\mu = m\frac{du^\mu}{d\tau} = m a^\mu$$

$$= m\frac{d}{d\tau}\left(\gamma(u)\begin{pmatrix} c \\ \vec{u}\end{pmatrix}\right)$$

$$= m\,\gamma(u)\frac{d}{d\tau}\begin{pmatrix} c \\ \vec{u}\end{pmatrix} + m\begin{pmatrix} c \\ \vec{u}\end{pmatrix}\frac{d}{d\tau}\gamma(u)$$

$$d\tau = \frac{dt}{\gamma}$$

$$= m\gamma^2\frac{d}{dt}\begin{pmatrix} c \\ \vec{u}\end{pmatrix} + m\begin{pmatrix} c \\ \vec{u}\end{pmatrix}\gamma\frac{d}{dt}\gamma$$

$$= m\gamma^2\begin{pmatrix} 0 \\ \vec{a}\end{pmatrix} + m\gamma^4\frac{\vec{v}\,\vec{a}}{c^2}\begin{pmatrix} c \\ \vec{u}\end{pmatrix}$$

vergleichen $\rightarrow$ $\vec{F}$ hat in den Termen um γ weniger

$$F^i = \gamma\,(\vec{F})^i \quad \nearrow \quad !!$$

Energiesatz

$$F^\mu = m a^\mu = \begin{pmatrix} F^0 \\ \gamma\vec{F}\end{pmatrix}$$

weg $a^\mu u_\mu = 0$

$$F^\mu u_\mu = \gamma\left(F^0 c - \gamma\vec{F}\vec{u}\right) = 0$$

$$\rightarrow F^0 = \frac{\gamma}{c}\vec{F}\vec{u} = \frac{\gamma}{c}\vec{F}\frac{\vec{u}\,dt}{dt} = \frac{\gamma}{c}\frac{\vec{F}\,d\vec{s}}{dt} = \frac{\gamma}{c}\frac{dA}{dt}$$

$$= \frac{d}{d\tau}\frac{A}{c}$$

$$F^0 = \frac{d}{d\tau}p^0 = \frac{d}{d\tau}m\gamma c$$

$$\rightarrow dA = d(m\gamma c^2)$$

Gradientenfeld: $\vec{F} = -\vec{\nabla}V$

$$dA = \vec{F}\,d\vec{s} = -\vec{\nabla}V\,d\vec{s} = -dV$$

$$\rightarrow d(m\gamma c^2) + dV = 0 \implies m\gamma c^2 + V = \underset{\text{konst}}{E} \checkmark$$

Freies Punktteilchen $\quad V=0$

$$\rightarrow E = \underbrace{m\gamma}_{m(u)} c^2 \qquad p^\mu = m u^\mu = m\gamma(u)\begin{pmatrix} c \\ \vec{u} \end{pmatrix}$$

$$\rightarrow p^\mu = \begin{pmatrix} E/c \\ \vec{p} \end{pmatrix}$$

$$\rightarrow p^\mu p_\mu = E^2/c^2 - \vec{p}^{\,2} = m^2 c^2$$

$$\rightarrow E = \sqrt{m^2c^4 + \vec{p}^{\,2}c^2} = mc^2\sqrt{1+\underbrace{\frac{\vec{p}^{\,2}}{m^2c^2}}_{a}}$$

nicht rel. Grenzfall $\quad v << c$

Reihe v $\sqrt{1+a} \qquad a \to \emptyset$

$$= 1 + \frac{1}{2}a - \frac{1}{8}a^2 \ldots$$

$$\Rightarrow E = \underbrace{mc^2}_{\text{Ruheenergie}} + \underbrace{\frac{1}{2}\frac{\vec{p}^{\,2}}{m}}_{E_{kin}} - \underbrace{\frac{1}{8}\frac{p^4}{m^3c^2}}_{\text{1. Korrekt. Term}}$$

Gleichf. beschl. Bewegung

Teilchen mit Ruhemasse M $\qquad dt=\emptyset \quad$ mit $\vec{F}=(F,0,0)$ "belastet"

$$\begin{pmatrix} F^0 \\ \gamma\vec{F} \end{pmatrix} = \frac{d}{d\tau}\begin{pmatrix} E/c \\ \vec{p} \end{pmatrix} = \gamma\frac{d}{dt}\begin{pmatrix} E/c \\ \vec{p} \end{pmatrix}$$

$$\vec{p} = M\gamma\vec{u} \qquad \rightarrow \vec{F} = \frac{d}{dt}M\gamma\vec{u} \qquad F \text{ in } x\text{-R}$$

$$\rightarrow F = \frac{d}{dt}M\gamma u \qquad Ft = M\gamma u$$

$$Ft = M\gamma\frac{dx}{dt}$$

$$F t = M \frac{1}{\sqrt{1-\frac{\dot{x}^2}{c^2}}}\,\dot{x} \qquad \Big| \,{}^2, \cdot \frac{1}{M}$$

$$\frac{F^2 t^2}{M^2} = \frac{\dot{x}^2}{1-\frac{\dot{x}^2}{c^2}}$$

$$\frac{F^2 t^2}{M^2}\left(1-\frac{\dot{x}^2}{c^2}\right) = \dot{x}^2$$

$$\frac{F^2 t^2}{M^2} = \dot{x}^2 + \frac{\dot{x}^2}{c^2}\,\frac{F^2 t^2}{M^2}$$

$$\frac{F^2 t^2}{M^2} = \dot{x}^2\left(1+\frac{F^2 t^2}{M^2 c^2}\right)$$

$$u^2 = \frac{\cancel{\frac{F^2}{M^2}}\, t^2}{\underbrace{1+\frac{F^2 t^2}{M^2c^2}}_{\frac{\cancel{F^2}}{\cancel{M^2}c^2}\left(\frac{M^2c^2}{F^2}+t^2\right)}} = \frac{c^2 t^2}{t^2 + \frac{M^2 c^2}{F^2}} \quad \checkmark$$

$$\frac{dx}{dt} = \frac{c t}{\sqrt{t^2 + \frac{M^2c^2}{F^2}}}$$

$$\int \frac{dx}{dt}\,dt = \int \frac{ct}{\sqrt{t^2+\frac{M^2c^2}{F^2}}}\,dt \qquad y = t^2 + c^2, \quad \frac{dy}{dt} = 2t$$

$$\longrightarrow \int \frac{c\cancel{t}}{2\cancel{t}\sqrt{y}}\,dy = \int \frac{c}{2}\, y^{-\frac{1}{2}}\,dy$$

$$= \frac{c}{\cancel{2}}\, y^{\frac{1}{2}} \cdot \cancel{2} + A$$

$$\Rightarrow \quad x(t) = c\sqrt{t^2 + \frac{M^2c^2}{F^2}} + A$$

$$= c\sqrt{\frac{M^2c^2}{F^2}\left(1+\frac{F^2t^2}{M^2c^2}\right)}$$

$$= \frac{Mc^2}{F}\sqrt{1+\frac{F^2t^2}{M^2c^2}} + A$$

$$x(0) = \frac{Mc^2}{F} + A = 0 \quad \rightarrow \quad A = -\frac{Mc^2}{F}$$

Berechnung der Eigenzeit

$$d\tau = \frac{dt}{\gamma} = dt\sqrt{1-\frac{u^2}{c^2}} = dt\sqrt{1-\frac{\cancel{c^2}t^2}{\cancel{c^2}\left(t^2+\frac{M^2c^2}{F^2}\right)}}$$

$$= dt\sqrt{\frac{\cancel{t^2}+\frac{M^2c^2}{F^2}-\cancel{t^2}}{t^2+\frac{M^2c^2}{F^2}}}$$

$$x = \frac{tF}{Mc}$$

$$d\tau = dt\,\frac{1}{\sqrt{1+\frac{t^2F^2}{M^2c^2}}}$$

$$\frac{dx}{dt} = \underbrace{\frac{F}{Mc}}_{a}$$

$$\tau = \int \frac{dx}{a}\frac{1}{\sqrt{1+x^2}} = \frac{\operatorname{arsinh} x}{a}$$

$$\rightarrow \tau = \operatorname{arsinh}\left(\frac{Ft}{Mc}\right)\frac{Mc}{F} \quad ✓$$

Teilchenstöße

$$p^\mu = \sum_A p^\mu_A$$

keine äußeren Kräfte → p^μ Erhaltungsgr.

$$p^\mu_{vor} = p^\mu_{nach}$$

ersetzt m, E & $\vec{p}$ Erhaltung

Systemwechsel

$$p'^\mu = \Lambda^\mu{}_\eta\, p^\eta$$

auch in S' gilt:

$$p'^\mu_{vor} = p'^\mu_{nach}$$

Schwerpunktsystem

System in dem räuml. k. v. $p^\mu_{ges} = \emptyset$ sind

S : kein SPS

4er Impuls ; $\vec{x}$ Achse in $\vec{p}$ drehen

$$p^\mu = \begin{pmatrix} E/c \\ p_0 \\ 0 \end{pmatrix}$$

$$p'^{\mu} = \Lambda^{\mu}{}_{\nu}\, p^{\nu} = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} E/c \\ p \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma E/c - \beta\gamma p \\ -\beta\gamma E/c + \gamma p \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\gamma}{c}(E - vp) \\ \frac{\gamma}{c}(-\beta E + pc) \\ 0 \\ 0 \end{pmatrix}$$

p'^{μ} ist SPS, wenn $\quad pc = \beta E$

$$\beta_s = \frac{cp}{E}$$

Einfach: $\quad p^{\mu} p_{\mu} = \frac{E^2}{c^2} - p^2 = p'^{\mu} p'_{\mu} = \frac{E'^2}{c^2}$ // Invariante Lorentz-Skalar

$$\to E' = \sqrt{E^2 - c^2 p^2}$$

$$\to p'^{\mu} = \begin{pmatrix} \sqrt{E^2 - c^2p^2}/c \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad ✓$$

oder Einsetzen (schwerer)

Lorentztrafo auf SPS

$$p^{\mu} p_{\mu} = c^2 M^2 = \underbrace{\frac{E^2}{c^2} - p^2}$$

Schwerpunktmasse

$$\Downarrow$$

$$c^4 M^2 = E^2 - p^2 c^2$$

mit $\beta_s = \frac{cp}{E}$

$$\to \gamma_s = \left(\sqrt{1 - \frac{c^2p^2}{E^2}}\right)^{-1} = \left(\frac{1}{E}\sqrt{E^2 - c^2p^2}\right)^{-1} = \left(\frac{c^2 M}{E}\right)^{-1}$$

$$= \frac{E}{c^2 M} \quad ✓$$

$$\beta_s \gamma_s = \frac{\cancel{c}p}{\cancel{E}} \frac{\cancel{E}}{c^{\cancel{2}} M} = \frac{p}{cM} \quad ✓$$ // Matrixelemente!

Einfacher: Inverse Lorentz-Trafo

$$\begin{pmatrix} \gamma & \beta\gamma & & \\ \beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} Mc \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} E/c \\ p \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma_s Mc \\ \beta_s \gamma_s Mc \\ 0 \\ 0 \end{pmatrix} \to \begin{matrix} \gamma_s \\ \beta_s \end{matrix}$$

Anwendung auf 2 Teilchen

x-Achse-Richtung Gesamtimpuls

$$p_A^\mu = \begin{pmatrix} E_A/c \\ p_A \\ p_x \\ p_y \end{pmatrix} \qquad p_B^\mu = \begin{pmatrix} E_B/c \\ p_B \\ -p_x \\ -p_y \end{pmatrix}$$

$$p^\mu = p_A^\mu + p_B^\mu = \begin{pmatrix} (E_A+E_B)/c \\ p_A+p_B \\ 0 \\ 0 \end{pmatrix}$$

$$m_A^2 c^2 = \frac{E_A^2}{c^2} - \vec{p}_A^{\,2} \qquad m_B^2 c^2 = \frac{E_B^2}{c^2} - \vec{p}_B^{\,2}$$

$$M^2 c^2 = \frac{(E_A+E_B)^2}{c^2} - (\vec{p}_A + \vec{p}_B)^2$$

$$M^2 c^2 = \underbrace{\frac{E_A^2}{c^2} - p_A^2}_{m_A^2 c^2} + \underbrace{\frac{E_B^2}{c^2} - p_B^2}_{m_B^2 c^2} + 2\left(\frac{E_A E_B}{c^2} - \vec{p}_A \vec{p}_B\right)$$

$$\rightarrow M^2 = (m_A + m_B)^2 + 2\left(\frac{E_A E_B}{c^4} - \frac{\vec{p}_A \vec{p}_B}{c^2} - m_A m_B\right) \checkmark$$

Elastische / inelastische Prozesse

elastisch, gleiche Anzahl & Aufbau der Teilchen

inelastisch: Änderung

Trotzdem Erhaltung d. Gesamtviererimpuls

elastischer Stoß zw. T.:

SPS:
$$p_A^\mu = \begin{pmatrix} E_A/c \\ \vec{p}_A \end{pmatrix} \qquad p_B^\mu = \begin{pmatrix} E_B/c \\ -\vec{p}_A \end{pmatrix}$$

nach Stoß:
$$p_{A'}^\mu = \begin{pmatrix} E_A'/c \\ \vec{p}_{A'} \end{pmatrix} \qquad p_{B'}^\mu = \begin{pmatrix} E_B'/c \\ -\vec{p}_{A'} \end{pmatrix}$$

Invar. v. $p^\mu p_\mu$: (Gesamtimpuls)

$$\left.\begin{aligned} E_A^2 - c^2 \vec{p}_A^{\,2} &= c^4 m_A^2 \\ E_B^2 - c^2 \vec{p}_A^{\,2} &= c^4 m_B^2 \end{aligned}\right\} \quad c^4(m_A^2 - m_B^2) = (E_A^2 - E_B^2)$$

nach Stoß analog

$$\left.\begin{aligned} E_{A'}^2 - c^2 \vec{p}_{A'}^{\,2} &= c^4 m_A^2 \\ E_{B'}^2 - c^2 \vec{p}_{B'}^{\,2} &= c^4 m_B^2 \end{aligned}\right\} \quad c^4(m_A^2 - m_B^2) = E_{A'}^2 - E_{B'}^2$$

$$\left.\begin{aligned} &\rightarrow \quad E_{A'} - E_{B'} = E_A - E_B \\ &\text{Gesamtenergieerhaltung} \quad E_{A'} + E_{B'} = E_A + E_B \end{aligned}\right] -$$

$-2E_{B'} = -2E_B$ ✓ $\qquad E_A = E_{A'}$

$E_B = E_{B'}$

$\rightarrow \qquad |\vec{p}_A| = |p_{A'}|$ aus Einzelgl.

reine Drehung d. Impulsvektoren

Elastischer Stoß von 2 Teilchen im Laborsys.

$$v_s^\mu = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix}$$ — im SPS !!

aus $E_A = E_{A'}$

wird $p_A^\mu v_{s\mu} = p_{A'}^\mu v_{s\mu}$ // Invar-Lorentz-Skalar

$$p_A^\mu = \begin{pmatrix} E_A/c \\ p_A \\ 0 \\ 0 \end{pmatrix} \qquad p_B^\mu = \begin{pmatrix} m_B c \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad p^\mu = \begin{pmatrix} (E_A + m_B c^2)/c \\ p_A \\ 0 \\ 0 \end{pmatrix}$$

$\rightarrow \quad v_s^\mu = \frac{1}{M} p^\mu$ — im CS !!

$p_A^\mu v_{s\mu} = p_{A'}^\mu v_{s\mu}$ $\qquad$ $p_B^\mu v_{s\mu} = p_{B'}^\mu v_{s\mu}$ $\qquad$ M ist egal

$\longrightarrow$ $p_A^\mu p_{s\mu}$

$$p_{A'}^\mu = \begin{pmatrix} E_{A'}/c \\ p_{A'}\cos\vartheta \\ p_{A'}\sin\vartheta \\ 0 \end{pmatrix} \qquad p_{B'}^\mu = \begin{pmatrix} E_{B'}/c \\ p_{B'}\cos\theta \\ p_{B'}\sin\theta \\ 0 \end{pmatrix}$$

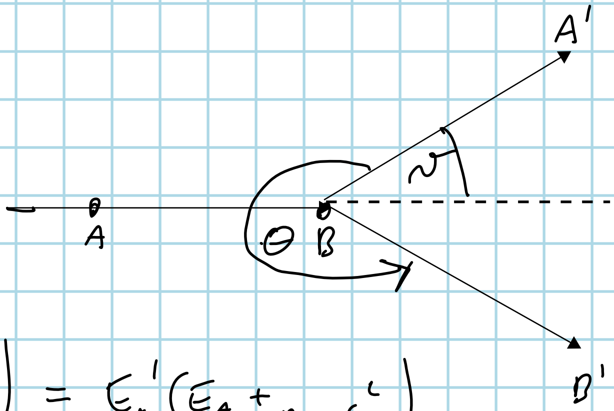
$$\frac{E_A}{c^2}\left(E_A + m_B c^2\right) - p_A^2 = \frac{E_{A'}}{c^2}\left(E_A + m_B c^2\right) - p_A p_A' \cos\vartheta \quad / \cdot c^2$$

$$\frac{m_B \cancel{c^2}}{\cancel{c^2}}\left(E_A + m_B c^2\right) = \frac{E_{B'}}{c^2}\left(E_A + m_B c^2\right) - p_A p_{B'}' \cos\theta \quad / \cdot c^2$$

$$\frac{E_{A'}^2}{c^2} - p_{A'}^2 = m_A^2 c^2 \qquad \text{\& f. B}$$

$\Rightarrow$ $E_A'(E_A, \vartheta)$

& $E_B'(E_A, \vartheta)$



$$E_A(E_A + m_B c^2) - c^2\left(\frac{E_A^2}{c^2} - m_A^2 c^2\right) = E_A'(E_A + m_B c^2)$$
$$- c^2 \sqrt{\frac{E_A^2}{c^2} - m_A^2 c^2}\sqrt{\frac{E_{A'}^2}{c^2} - m_A^2 c^2}\,\cos\vartheta$$

2. Komp. von p^μ

& $p^\mu = p^{\mu'}$ $\longrightarrow$ $p_A' \sin\vartheta + p_B' \sin\theta = \emptyset$

1. Teilch. Winkel $\qquad$ 2. T. Winkel

Compton – Streuung

Photon $\rightarrow$ o e^- $\qquad$ $c = \lambda\nu$

$$E = \hbar\omega = h\nu = \frac{hc}{\lambda}$$

Photon: $\emptyset = \frac{E_A^2}{c^2} - p_A^2$

auch nach Stoß

p bleibt wie beim Stoß zw. T im CS
e^- ruht

$$c^2 p_A^2 = E_A^2$$

$$E_A(E_A + m_e c^2) - E_A^2 = E_A'(E_A + m_e c^2) - E_A E_A' \cos\vartheta$$

$$/ : E_A E_A' m_e c^2$$

$$\rightarrow \quad \frac{1}{E_A'} = \underbrace{\frac{1}{m_e c^2} + \frac{1}{E_A} - \frac{\cos\vartheta}{m_e c^2}}_{\frac{1}{m_e c^2}(1-\cos\vartheta) + \frac{1}{E_A}}$$

$$\rightarrow \quad E = \frac{hc}{\lambda}$$

$$\frac{\lambda_A'}{hc} = \frac{1}{m_e c^2}(1-\cos\vartheta) + \frac{\lambda_A}{hc} \quad / \cdot hc$$

$$\lambda_A' = \underbrace{\frac{hc}{m_e c^2}}_{\lambda_C}(1-\underbrace{\cos\vartheta}_{\text{Streuwinkel}}) + \lambda_A$$

ϑ : Winkel d. Photons

Inelastische Proton / Proton Streuung

$$\underset{\text{bew.}}{p^+} + \underset{\text{ruht}}{p^+} \rightarrow p^+ + p^+ + p^+ + p^-$$

$$p^+_{\text{bew}} = \begin{pmatrix} E/c \\ \vec{p} \end{pmatrix} \quad p^+_{\text{ruhe}} = \begin{pmatrix} mc \\ \vec{0} \end{pmatrix}$$

$$p^\mu = \begin{pmatrix} (E + mc^2)/c \\ \vec{p} \end{pmatrix}$$

p- bew.: $\frac{E^2}{c^2} - \vec{p}^{\,2} = m^2c^2$

nach Stoß im SPS $\qquad p'^\mu_{\text{nachher}} = \begin{pmatrix} 4mc \\ \vec{\emptyset} \end{pmatrix}$

Invarianz d. Vierervektors:

$\vec{p}$ vor dem Zerf.

$$(E + mc^2)^2 - c^2\vec{p}^2 = (4mc)^2 c^2$$

$$\cancel{E^2} + 2Emc^2 + m^2c^4 - \cancel{E^2} + m^2c^4 = 16m^2c^4$$

$$\cancel{2}E\cancel{m}\cancel{c^2} = \cancel{14}_{7}\, m^{\cancel{2}}c^{\cancel{4}\,2}$$

$$E = 7mc^2$$

$$\longrightarrow \quad \gamma = 7 \quad \longrightarrow \quad v = \sqrt{\tfrac{48}{49}}\, c$$

Zerfall eines Photons

$$\gamma \rightarrow e^- + e^+ \;?$$

$$p_\gamma^\mu = \begin{pmatrix} E/c \\ \vec{p} \end{pmatrix} \quad p_{e-}^\mu = \begin{pmatrix} E_A'/c \\ \vec{p}_A' \end{pmatrix} \quad p_{e+}^\mu = \begin{pmatrix} E_B'/c \\ \vec{p}_B' \end{pmatrix}$$

Gesamtvierervektor:

$$p_\gamma^\mu = p_{e-}^\mu + p_{e+}^\mu$$

$$\begin{pmatrix} E/c \\ \vec{p} \end{pmatrix} = \begin{pmatrix} (E_A' + E_B')/c \\ \vec{p}_A + \vec{p}_B \end{pmatrix}$$

$$p^\mu p_\mu = \underbrace{m^2c^2}_{\emptyset \text{ bei Photon}}$$

$$\longrightarrow \quad (E_A' + E_B')^2 - c^2(\vec{p}_A' + \vec{p}_B')^2 = \emptyset$$

Energie Impuls - Rel:

$$E^2 - \vec{p}c^2 = \emptyset \qquad E_{A'}^2 - p_A'^2c^2 = m^2c^4$$

$$E_{B'}^2 - p_B'^2c^2 = m^2c^4$$

$$\longrightarrow \quad E_A'^2 + 2E_A'E_B' + E_B'^2 - c^2(\vec{p}_{A'}^{\,2} + 2\vec{p}_{A'}\vec{p}_{B'} + p_{B'}^2) = \emptyset$$

$$\rightarrow 2m^2c^4 + p_A'^2c^2 + 2\sqrt{m^2c^4+p_A'^2c^2}\sqrt{m^2c^4+p_B'^2c^2} + p_B'^2c^2 - c^2(\ldots) = 0$$

$$2m^2c^4 + \underbrace{2\sqrt{}\sqrt{} - 2c^2\,\vec{p}_A'\,\vec{p}_B'}_{>0} = 0$$

$\rightarrow$ unerfüllbar!